Finding the optimal way to repay student debt
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The shaded areas denote situations in which income-based payments would minimize the cost for direct undergraduate, graduate, and PLUS loans, according to the time until loan forgiveness in an income-based scheme (on the vertical axis, in years) and loan balance (on the horizontal axis, in U.S. dollars). The light gray area represents immediate enrollment and the dark gray area represents later enrollment after a period of maximum payments. In areas with no shading, the optimal choice is to make maximum payments until the loan is fully paid off. The vertical lines denote the maximum loan amounts that are currently allowed for those schemes, indicating that it is always best to make maximum payments on direct undergraduate loans and often best to do so for direct graduate loans as well. Credit: Paolo Guasoni, Yu-Jui Huang, and Saeed Khalili.

In a paper that published in April in the SIAM Journal on Financial Mathematics, Guasoni—along with Yu-Jui Huang and Saeed Khalili (both of the University of Colorado, Boulder)—developed a strategy for minimizing the overall cost of repaying student loans. "In the literature, we found mostly empirical studies discussing what borrowers are doing," Huang said. "But what we wanted to know was rather, how should a borrower repay to minimize debt burden?"

Students become responsible for repaying their loans a few months after they graduate or unenroll, and must contend with the loan growing at a national fixed interest rate. One option for borrowers is to repay their balances in full by a 'fixed maturity'—the date at which a loan's final payment is due. Another is to enroll in an income-based scheme, in which monthly payments are only due if the borrower has an income above a certain subsistence threshold. If payments are required, they are proportional to the amount the borrower makes above that threshold. After roughly 20 to 25 years, any remaining balance is forgiven but taxed as ordinary income. "The tension is between postponing payments until forgiveness and letting interest swell the loan balance over time," Guasoni said. The tax cost of delaying payments increases exponentially with longer timeframes until forgiveness, potentially offsetting the supposed savings.

The intuitive approach for many borrowers may be to pay off small loans as quickly as possible, since even minimum payments would extinguish the balance by the end of its term, making forgiveness irrelevant. Similarly, one may wish to minimize the payments for a large loan through an income-based scheme, especially if the loan will be forgiven in a few years anyway. However, the situation is not always as simple as it seems. "The counterintuitive part is that, if your loan is large and forgiveness is far away, it may be better to maximize payments over the first few years to keep the loan balance
from exploding," Huang said. "Then you can switch to income-based repayment and take advantage of forgiveness."

To investigate what is truly the optimal way to pay back a student loan, the authors created a mathematical model of a borrower who took out a federal student loan—the most common type of student loan—with a constant interest rate. The model assumes that the borrower is able to repay the loan under its original term and even possibly make additional payments; otherwise, they would have no choice but to enroll in an income-based scheme. Quickly paying off the loan leads to lower costs from compounding interest. However, the borrower's motivation to do so is contradicted by the possibility of the remaining balance being forgiven and taxed in the future, which encourages them to delay payment until the forgiveness date.

The mathematical model revealed several possible approaches for a borrower who wishes to minimize the overall cost of their loan. "The optimal strategy is to either (i) repay the loan as quickly as possible [if the initial balance is sufficiently low], or (ii) maximize payments up to a 'critical horizon' (possibly now) and then minimize them through income-based repayment," Guasoni said. The critical horizon occurs when the benefits of forgiveness begin to outweigh the compounding costs of interest on the loan balance. For large loans with a high interest rate—which are common for professional degrees—the savings from the strategy of high initial payments followed by enrollment in an income-based scheme can be substantial, for those that are able to afford such a plan.

The authors provided an example of a dental school graduate with a balance of $300,000 in Direct PLUS loans that carry an interest rate of 7.08 percent (according to the American Dental Education Association, 83 percent of dental school graduates have student loan debt, with an average balance of $292,169). This graduate has a starting salary of $100,000 that will grow four percent annually, and is able to repay at most 30 percent of the income that they make above the subsistence level. If they kept up such maximal payments, they would repay the loan in less than 20 years with a total cost of $512,000.

The example graduate could also immediately enroll in income-based repayment, paying only 10 percent of the income that they make above subsistence. After 25 years, their balance would equal $1,053,000 due to compounding interest. This balance would be forgiven and taxed as income at a 40 percent rate, yielding a total cost of $524,000. Alternatively, the graduate could use the authors' suggested strategy and repay 30 percent of their income above subsistence for around nine years, then switch to the income-based repayment scheme. The remaining balance to be forgiven after 25 years would then be $462,000, leading to a total cost from payments and tax of $490,000—the lowest of all the strategies. The reduction in the balance through multiple years of high payments curbs the balance's ensuing growth during the period of minimum payments.

Future research could further explore the more complicated factors of student debt repayment. The authors' model is deterministic—it does not account for the fact that the interest rates could potentially change in the future. However, interest rates can increase or decrease, which may compel borrowers to refinance or delay payments. Further work is necessary to determine the influence of such changes on optimal debt repayment.

This research illuminated the way in which borrowers' choices in their loan repayments can have a sizable impact on overall costs, especially given compounding interest. "If you have student loans, you should consider your specific options carefully and see what the total cost would be under different strategies," Guasoni said. Huang agreed, noting that their proposed strategy may be especially beneficial for the large loans that are often held by law and dental school graduates. "Each loan is slightly different," he said. "Our model does not capture every possible detail, but it helps to focus the attention on two possibilities: quickest full repayment or enrollment in an income-based scheme, possibly after a period of high payments."

A careful, mathematical consideration of the approach to loan repayment can help borrowers make decisions that will benefit them in the years to come.