The discrepancy between mathematical proofs, algorithms, and their implementations in control systems

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Engineers work in quantifiable realism—an object exists and can be measured. Sometimes, though, the certainty of the object and how it will behave wavers. Researchers from the Automatic Control and System Dynamics Laboratory at the Technische Universität Chemnitz in Germany are starting to close the gap between reality and mathematical uncertainty.

They published an analysis of the discrepancy between mathematical proofs, algorithms, and their implementations in control systems with real, measurable outcomes. Their work appears in the July issue of *IEEE/CAA Journal of Automatica Sinica* (*JAS*), a joint publication of the IEEE and the Chinese Association of Automation.

"Control systems appear in everything from washing machines to rockets," said Pavel Osinenko, an author on the paper. "Control engineers work with objects that correspond with reality. For models of real objects, we need to develop real controllers that work in the final application. Classical mathematics are good to investigate highly abstract objects, but they overshoot on control theory."

In classical mathematic theory, Osinenko said, strength is an important factor that can miss the point of control theory. Strength, in this case, refers to the specificity of the information conveyed. Some mammals are humans, and some humans are women, and some women are mothers. In classical mathematics, it's stronger to know a variable in an equation is a human mother than simply a mammal, because more information can be inferred.

"In order for control theory to work, it requires a logical background that is way weaker," Osinenko said, noting that classical mathematics requires a logical system of several steps to ensure the most specific information to stay as strong as possible. "We need a minimalistic logical system for control theory."

The researchers analyzed a hundred-year-old theorem by mathematician Constantin Carathéodory. The theorem purports that a problem with a changeable independent variable, such as the trajectory of a thrown ball, can be solved with weak logical systems.

"It's constructive mathematics—every object that you can construct or prove to exist is computable. You can input a mathematical proof one to one in your computer," Osinenko said.

That's not the case in classical mathematics where objects are often proven by assuming they don't exist until contradictory mathematics provide evidence.

The researcher explored a variant of Caratheordory's theorem that covers several problems in practice and not just in theory. It's the link between theorems and proofs and computational certainty.

"Classical mathematics says there's a black cat in a dark room. It's definitely in there, but you can't point to its precise location," Osinenko said. "This minimal logical system is the torch with which we light up the room. The cat is right there."

The authors plan to further investigate minimal logic systems and constructive mathematics, with a focus on automated reasoning to aid in solutions for control systems.

"There's an ocean of mathematical results and theories in control theory that still wait for their constructive treatment," Osinenko said. "The next step is for us to pick one and work it out."

More information: Pavel Osinenko et al, Analysis of the Caratheodory's theorem on dynamical system trajectories under numerical uncertainty, *IEEE/CAA Journal of Automatica Sinica* (2018). DOI: 10.1109/JAS.2018.7511135

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