When inertial frames of reference collide

June 7 2018, by Christopher Packham


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GALILEO GALILEI

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In an inertial frame of reference, a body with zero net force acting on it does not accelerate. When scientists speak of inertial frames of reference, they're invoking a coordinate system with no external influences, and which describes space and time homogeneously and with uniformity in all directions. This was Galileo's clever conceptual solution to the problem of describing inertial systems mathematically.

The laws of motion are exactly the same in all frames, which is the basis of the Galilean invariance principle—that is to say, the laws of physics do not vary between frames. Additionally, all frames of reference are in a constant state of motion with respect to all other frames of reference, and measurements in one frame can be converted to measurements in another frame by means of a simple transformation. These transformations preserve time intervals and distances between simultaneous events.

The problem is that real-world systems are described via coarse-grained models that integrate variables including friction and stochastic processes that serve as models of phenomena that appear to vary randomly. And including them in a coarse-grained real-world model has the unfortunate effect of violating Galilean invariance.

Andrea Cairoli from Imperial College London and collaborators have now published a paper in the *Proceedings of the National Academy of Sciences* that shows how Galilean invariance becomes broken in such models when deriving stochastic equations, and provides a solution to this problem. They studied the coarse-graining process in different frames and determined that stochastic models cannot be chosen based on their correspondence with the data alone—to preserve physical consistency between reference frames, they also have to satisfy another invariance principle, which the researchers have termed "weak Galilean invariance."

Here's the problem: Consider anomalous diffusion, a complex stochastic process with a nonlinear relationship to time. The authors point out that anomalous diffusion has been observed in wide range of physical processes, including charge transport in semiconductors, particle transport in plasmas, the intracellular transport of mitochondria, and the intracellular behavior of lipid and insulin granules. Due to the intrinsic difficulties of assessing complex microscopic interactions in such experiments, theoretical models for these phenomena can't be derived from first principles. So there is no fundamental rule associated with anomalous diffusion that can be used to verify the physical consistency of such models between frames and thus satisfy Galilean invariance.

Galilean invariance is debated with regard to the derivation of the Navier-Stokes equations related to fluid dynamics, and invariance is equally contentious for the Kardar-Parisi-Zhang equation, which is a nonlinear stochastic partial differential equation. The paper establishes that stochastic, coarse-grained descriptions including them violate Galilean invariance, but describes in detail a conjecture that includes three important properties required to satisfy weak Galilean invariance.

The authors write, "Our most important statement is that ignoring our weak Galilean invariance rules can easily lead to unphysical models… The consequences of our results are thus far-reaching. Weak Galilean invariance is expected to constrain all mesoscopic diffusive models whose microscopic representation is expected to satisfy conventional Galilean invariance." The authors add that their findings have far-reaching application in modeling approaches for physical, chemical and biological processes.

More information: Weak Galilean invariance as a selection principle for coarse-grained diffusive models. *Proceedings of the National Academy of Sciences* DOI: 10.1073/pnas.1717292115

Abstract
How does the mathematical description of a system change in different reference frames? Galilei first addressed this fundamental question by formulating the famous principle of Galilean invariance. It prescribes that the equations of motion of closed systems remain the same in different inertial frames related by Galilean transformations, thus imposing strong constraints on the dynamical rules. However, real world systems are often described by coarse-grained models integrating complex internal and external interactions indistinguishably as friction and stochastic forces. Since Galilean invariance is then violated, there is seemingly no alternative principle to assess a priori the physical consistency of a given stochastic model in different inertial frames. Here, starting from the Kac–Zwanzig Hamiltonian model generating Brownian motion, we show how Galilean invariance is broken during the coarse-graining procedure when deriving stochastic equations. Our analysis leads to a set of rules characterizing systems in different inertial frames that have to be satisfied by general stochastic models, which we call "weak Galilean invariance." Several well-known stochastic processes are invariant in these terms, except the continuous-time random walk for which we derive the correct invariant description. Our results are particularly relevant for the modeling of biological systems, as they provide a theoretical principle to select physically consistent stochastic models before a validation against experimental data.