Systematic onset of periodic patterns in random disk packings

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Physicists at Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU) have proven that random packings of disks of the same size between parallel walls always form a periodic structure, regardless of the width of the container. The results, which should help scientists to better understand the packing properties of microparticles, have now been published in the renowned journal *Physical Review Letters*.

People have wondered which patterns should be used to pack objects at the highest density in containers for several centuries now. As early as 1611, Johannes Kepler suggested that no arrangement of spheres of the same size had a greater density than offset layers in hexagonal lattices. While packings of spheres randomly poured into a box have a mean density of around 65 percent, a density of around 74 percent can be achieved with the periodic structure of hexagonal packing. Kepler's theory was finally proven in 2014 by using complex computer simulations.

In conjunction with colleagues from Brazil and the USA, physicists at FAU have now discovered that when spheres are poured randomly into a container, they always form a periodic structure. They were able to confirm this with two-dimensional experiments. In a series of computer simulations, the researchers poured up to 10 million disks of the same size from various positions into an open rectangular container. The researchers were astonished to find that a periodic structure formed during every single one of the simulations. "In our case, periodic means that there are equivalents for each particle that are repeated at the same position on the x-axis at regular intervals," explains Prof. Dr. Thorsten Pöschel from the Institute for Multiscale Simulation of Particulate Systems at FAU. The pattern of disks and voids that forms continues upwards in a uniform fashion with an average of four contacts per disk.

However, these periodic patterns do not form immediately. Initially, there is a disordered phase that is mainly characterised by larger spaces or by clusters of disks that have more or less than four contacts. Although the fill level at which the disks form a periodic structure can vary greatly between containers of the same width, this average level increases when the distance between the walls of the container increases. Or, to put it another way, the wider the channel, the more layers have to be poured in until the disks form periodic patterns. This is because there are more ways for the disks to arrange themselves in disordered positions at the beginning of the filling process, and this continues upward in significantly more layers than in narrow containers. But regardless of the width of the container, the researchers were able to demonstrate that the probability that a channel is not yet periodic decreases exponentially with increasing fill level.

The findings should help to increase the understanding of packing properties of monodisperse and polydisperse microparticles. Packing particles as densely as possible is the key to several practical applications, for example, for minimising material porosity during 3-D printing processes and other methods of additive manufacturing, thus increasing the strength of new materials.

The results of the project have now been published in *Physical Review Letters*, titled "Systematic Onset of Periodic Patterns in Random Disk Packings."

More information: Nikola Topic et al, Systematic Onset of Periodic Patterns in Random Disk Packings, *Physical Review Letters* (2018). DOI: 10.1103/PhysRevLett.120.148002

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