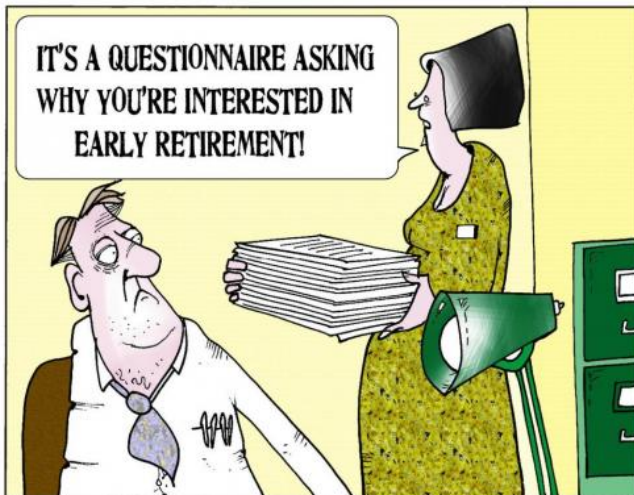


Are you ready to retire? Mathematical models estimate the value of pension plans

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This cartoon relates to pensions and early retirement.
Credit: (c) Fran, Jantoo.com

There comes a time in each of our lives when we consider starting a pension plan –either on the advice of a friend, a relative, or of our own volition. The plan of choice may depend on various factors, such as the age and salary of the individual, number of years of expected employment, as well as options to retire early or late.

One possible plan is a defined [pension plan](#), where the benefit amount is typically based on the employee's number of years of service at the time of retirement and the [salary](#) and/or average salary over an employment period. For instance, the employee may receive a fraction of the average salary during a certain number of years.

In a paper published last month in the *SIAM Journal on Applied Mathematics*, authors Carmen Calvo-Garrido, Andrea Pascucci, and Carlos Vázquez present a partial differential equation (PDE) [model](#) governing the value of a defined pension plan including the option for early

retirement.

"The employer bears the liability of the pension and the value of this liability is understood as the value of the pension plan," says author Carlos Vázquez. "It is important to develop mathematical models to compute the value of this liability in order to estimate the financial situation of the institution or company that has the obligation with the pension plan member."

The analysis in the paper uses [modeling tools](#) similar to those used in quantitative finance, for instance, for pricing American options.

The model assumes that the wage or salary of an employee at any given time is governed by a stochastic [differential equation](#), which in turn depends on the time of recruitment, current salary of the employee and age of entry. Uncertainty of the salary is assumed to depend only on volatility, which refers to the uncertainty or risk associated with a value or asset. "Models need to reproduce the uncertainties associated with the underlying factors of the plan (salary, interest rate and so on) and should allow one to compute the pension plan price in order to reproduce situations in different scenarios," author Andrea Pascucci explains.

The authors obtain the value of a defined benefit pension plan including the option for early retirement for the employee, thus computing the value of the pension plan as well as the region of early retirement. "If the pension plan incorporates the option of [early retirement](#) by the member, then the additional question arises: when is it optimal to retire? Mathematical modeling tools allow us to pose the problem in terms of partial differential equations," says Vázquez.

The optimal retirement problem is a "free boundary problem" for the underlying PDE. Most applications of PDEs involve domains with boundaries, and certain boundary conditions need to be satisfied in

order to solve the PDEs. Free boundary problems deal with solving PDEs where part of the boundary is unknown in advance, referred to as a free boundary. Thus, in addition to standard boundary conditions, an additional condition must be imposed at the free boundary. The free boundary in this problem is the optimal retirement boundary between the region where it is optimal to retire and the region where it is optimal to continue working.

"The practical solution of the PDE model to obtain pension plan prices from the data requires the use of suitable numerical algorithms to be run on a computer," says author M. Carmen Calvo-Garrido. "From the numerical solutions, we can identify at each date, for a given salary and average salary, if it is optimal to retire or not, and also to obtain the value of the pension plan in any case."

Mathematical analysis provides rigorous justification of the correctness of the model, also proving the expected qualitative properties.

Future directions may involve the application of similar modeling techniques to study the evolution of wages and salaries. "We are working on a more complete model for salaries evolution that includes the possibility of jumps (due to economic crisis, sudden increase or decrease in salaries, etc)," says Vázquez. "PDE problems including realistic, stochastic interest rate models also present a very challenging topic. The calibration of model parameters is an interesting and difficult problem due to the need of suitable real data."

More information: Mathematical Analysis and Numerical Methods for Pricing Pension Plans Allowing Early Retirement, M. Carmen Calvo-Garrido, Andrea Pascucci, and Carlos Vázquez, *SIAM Journal on Applied Mathematics*, 73(5), 1747-1767 (Online publish date: September 4, 2013). epubs.siam.org/doi/abs/10.1137/120864751

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