Is mathematics an effective way to describe the world?

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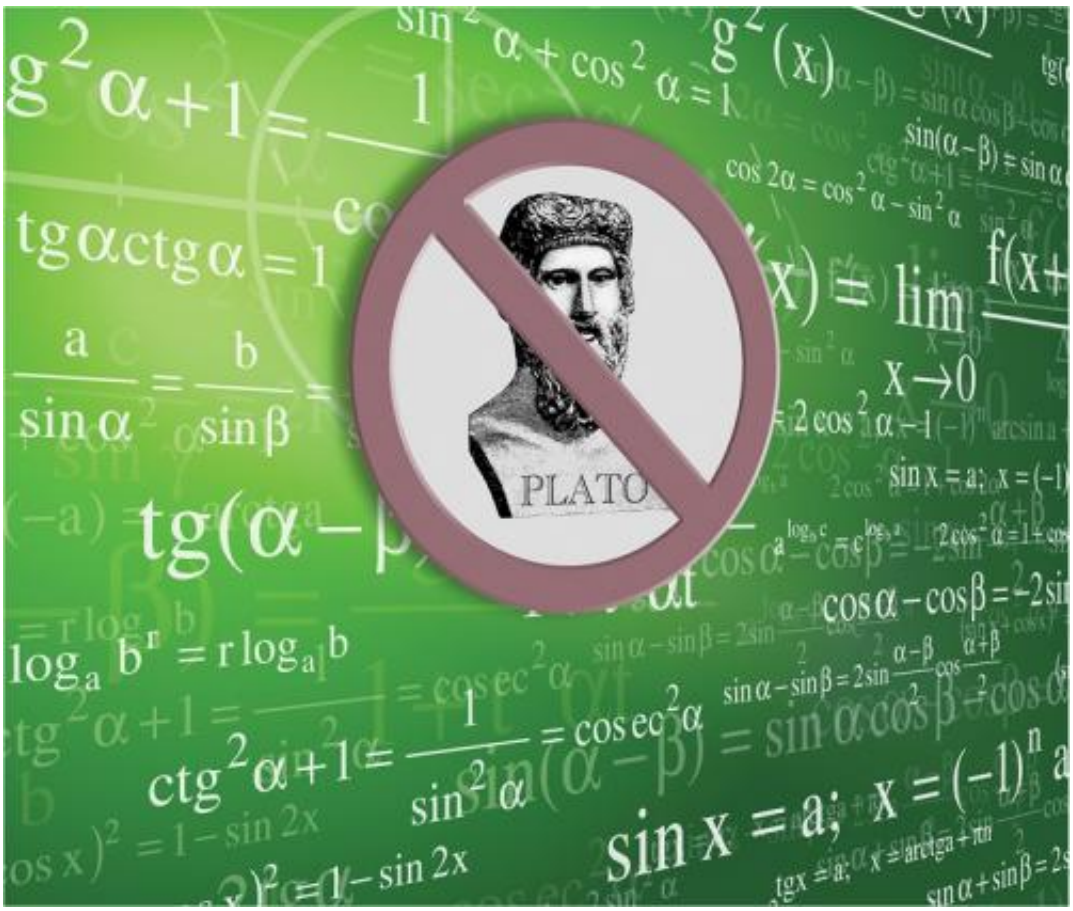

Math has the illusion of being effective when we focus on the successful examples, Abbott argues. But there are many more cases where math is ineffective than where it is effective. Credit: Derek Abbott. ©2013 IEEE

Mathematics has been called the language of the universe. Scientists and

engineers often speak of the elegance of mathematics when describing physical reality, citing examples such as π, $E=mc^2$, and even something as simple as using abstract integers to count real-world objects. Yet while these examples demonstrate how useful math can be for us, does it mean that the physical world naturally follows the rules of mathematics as its "mother tongue," and that this mathematics has its own existence that is out there waiting to be discovered? This point of view on the nature of the relationship between mathematics and the physical world is called Platonism, but not everyone agrees with it.

Derek Abbott, Professor of Electrical and Electronics Engineering at The University of Adelaide in Australia, has written a perspective piece to be published in the *Proceedings of the IEEE* in which he argues that mathematical Platonism is an inaccurate view of reality. Instead, he argues for the opposing viewpoint, the non-Platonist notion that mathematics is a product of the human imagination that we tailor to describe reality.

This argument is not new. In fact, Abbott estimates (through his own experiences, in an admittedly non-scientific survey) that while 80% of mathematicians lean toward a Platonist view, engineers by and large are non-Platonist. Physicists tend to be "closeted non-Platonists," he says, meaning they often appear Platonist in public. But when pressed in private, he says he can "often extract a non-Platonist confession."

So if mathematicians, engineers, and physicists can all manage to perform their work despite differences in opinion on this philosophical subject, why does the true nature of mathematics in its relation to the physical world really matter?

The reason, Abbott says, is that because when you recognize that math is just a mental construct—just an approximation of reality that has its frailties and limitations and that will break down at some point because

perfect mathematical forms do not exist in the physical universe—then you can see how ineffective math is.

And that is Abbott's main point (and most controversial one): that mathematics is not exceptionally good at describing reality, and definitely not the "miracle" that some scientists have marveled at. Einstein, a mathematical non-Platonist, was one scientist who marveled at the power of mathematics. He asked, "How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?"

In 1959, the physicist and mathematician Eugene Wigner described this problem as "the unreasonable effectiveness of mathematics." In response, Abbott's paper is called "The Reasonable Ineffectiveness of Mathematics." Both viewpoints are based on the non-Platonist idea that math is a human invention. But whereas Wigner and Einstein might be considered mathematical optimists who noticed all the ways that mathematics closely describes reality, Abbott pessimistically points out that these mathematical models almost always fall short.

What exactly does "effective mathematics" look like? Abbott explains that effective mathematics provides compact, idealized representations of the inherently noisy physical world.

"Analytical mathematical expressions are a way making compact descriptions of our observations," he told *Phys.org*. "As humans, we search for this 'compression' that math gives us because we have limited brain power. Maths is effective when it delivers simple, compact expressions that we can apply with regularity to many situations. It is ineffective when it fails to deliver that elegant compactness. It is that compactness that makes it useful/practical ... if we can get that compression without sacrificing too much precision.

"I argue that there are many more cases where math is ineffective (non-compact) than when it is effective (compact). Math only has the illusion of being effective when we focus on the successful examples. But our successful examples perhaps only apply to a tiny portion of all the possible questions we could ask about the universe."

Some of the arguments in Abbott's paper are based on the ideas of the mathematician Richard W. Hamming, who in 1980 identified four reasons why mathematics should not be as effective as it seems. Although Hamming resigned himself to the idea that mathematics is unreasonably effective, Abbott shows that Hamming's reasons actually support non-Platonism given a reduced level of mathematical effectiveness.

Here are a few of Abbott's reasons for why mathematics is reasonably ineffective, which are largely based on the non-Platonist viewpoint that math is a human invention:

• Mathematics appears to be successful because we cherry-pick the problems for which we have found a way to apply mathematics. There have likely been millions of failed mathematical models, but nobody pays attention to them. ("A genius," Abbott writes, "is merely one who has a great idea, but has the common sense to keep quiet about his other thousand insane thoughts.")

• Our application of mathematics changes at different scales. For example, in the 1970s when transistor lengths were on the order of micrometers, engineers could describe transistor behavior using elegant equations. Today's submicrometer transistors involve complicated effects that the earlier models neglected, so engineers have turned to computer simulation software to model smaller transistors. A more effective formula would describe transistors at all scales, but such a compact formula does not exist.

• Although our models appear to apply to all timescales, we perhaps create descriptions biased by the length of our human lifespans. For example, we see the Sun as an energy source for our planet, but if the human lifespan were as long as the universe, perhaps the Sun would appear to be a short-lived fluctuation that rapidly brings our planet into thermal equilibrium with itself as it "blasts" into a red giant. From this perspective, the Earth is not extracting useful net energy from the Sun.

• Even counting has its limits. When counting bananas, for example, at some point the number of bananas will be so large that the gravitational pull of all the bananas draws them into a black hole. At some point, we can no longer rely on numbers to count.

• And what about the concept of integers in the first place? That is, where does one banana end and the next begin? While we think we know visually, we do not have a formal mathematical definition. To take this to its logical extreme, if humans were not solid but gaseous and lived in the clouds, counting discrete objects would not be so obvious. Thus axioms based on the notion of simple counting are not innate to our universe, but are a human construct. There is then no guarantee that the mathematical descriptions we create will be universally applicable.

For Abbott, these points and many others that he makes in his paper show that mathematics is not a miraculous discovery that fits reality with incomprehensible regularity. In the end, mathematics is a human invention that is useful, limited, and works about as well as expected.

For those who seek something more practical out of such a discussion, Abbott explains that this understanding can allow for greater freedom of thought. One example is an improvement of vector operations. The current method involves dot and cross products, "a rather clunky" tool that does not generalize to higher dimensions. Lately there has been a renewed interest in an alternative approach called geometric algebra,

which overcomes many of the limitations of dot and cross products and can be extended to higher dimensions. Abbott is currently working on a tutorial paper on geometric algebra for electrical engineers to be published in the near future.

More information: More information: Derek Abbott. "The Reasonable Ineffectiveness of Mathematics." *Proceedings of the IEEE.* To be published. DOI: 10.1109/JPROC.2013.2274907