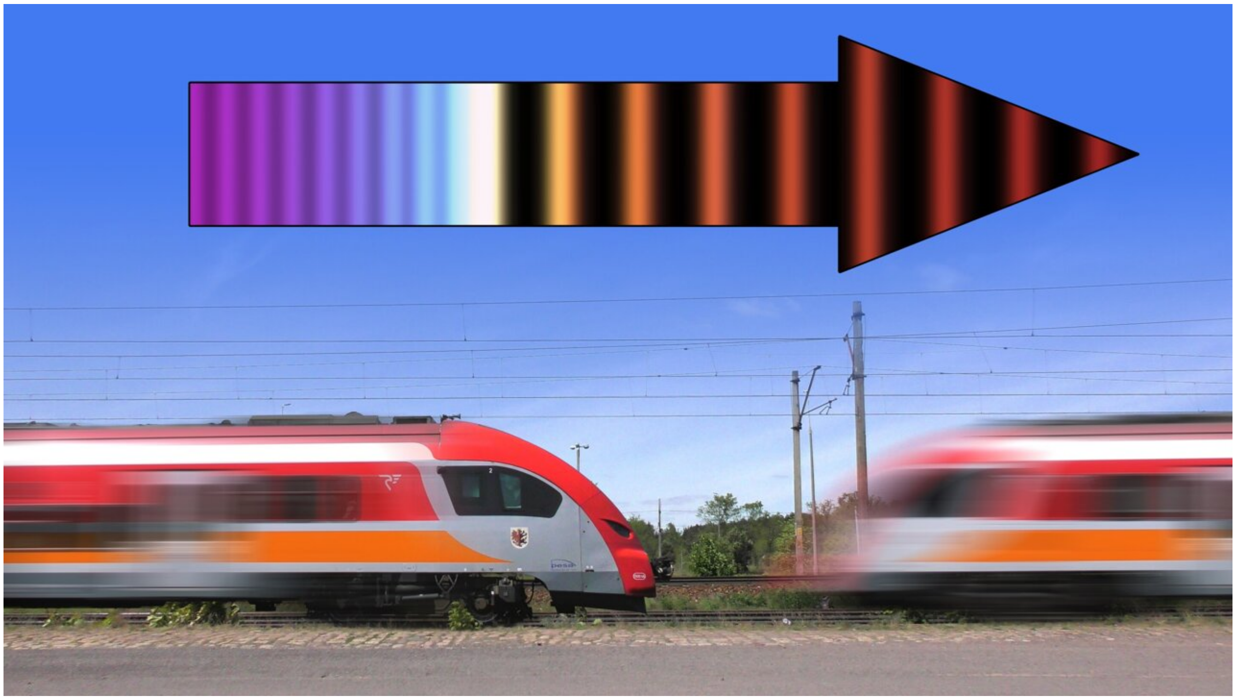
PHYS.ORG

From burns to the wave nature of heat—via the telegraph equation

May 23 2024



The Doppler effect can be heard particularly clearly when a train passes by. The presence of the same effect in the generalized telegraph equation indicates the wave nature of heat transport over small distances. Credit: IFJ PAN

When a train approaches or an ambulance with its siren blaring nears us, we hear the sound with an increased frequency, gradually decreasing slightly. As it passes, the frequency changes abruptly to a lower one, then decreases further. This commonly encountered Doppler effect can be a valuable clue to the nature of a phenomenon seemingly completely unrelated to sound propagation: heat transport.

Burns are not pleasant for anyone, but they affect physicists two-fold: not only do they suffer in the normal way, in addition, they still do not know which mechanism is responsible for heat transport in systems as complex as biological tissues.

Is it diffusion, associated with the spread of initially clustered molecules of matter? Or are wave phenomena similar to those known from acoustics responsible for heat transport?

A group of three theoreticians from the Institute of Nuclear Physics of the Polish Academy of Sciences (IFJ PAN) in Cracow decided to tackle the problem of heat transport using the telegraph equation and the Doppler effect, well known to us from everyday life (and primary school). The results of the team's work have just been published in the *International Journal of Heat and Mass Transfer*.

In physics, wave motion is described by an equation called the wave equation. When telegraph technology was developing in the second half of the 19th century, it became apparent that, to describe a message transmitted in Morse code, this equation had to be modified to take into account the attenuation of the current flowing through the medium in which it propagates, i.e. through the telegraph cable.

With telecommunications in mind, the telegraph equation was then developed to describe how electric current propagates with attenuation along one spatial dimension.

"In recent years, the skillfully generalized telegraph equation has found a new application: it has also started to be used to describe phenomena related to diffusion or heat transport. This fact spurred us on to pose an intriguing question," says Dr. Katarzyna Gorska (IFJ PAN).

"In solutions of the wave equation, i.e. without damping, the Doppler effect occurs. This is a typical wave phenomenon. But does it also occur in solutions of telegraph equations related to heat transport? If so, we would have an excellent indication that, at least from the theoretical point of view, there is no reason to believe that in systems with damping—for example, in biological tissue—heat flow could not be treated as a wave phenomenon."

The classical Doppler effect is the apparent change in frequency of waves emitted by a source moving relative to an observer. When the distance between the source and the observer decreases, the maxima and minima of the emitted waves reach the receiver more frequently than when the distance between the source and the observer increases. In the case of sound waves, we can clearly hear that the sound of an approaching train or the siren of a fast approaching ambulance have noticeably higher frequencies than when these vehicles are moving away from us.

Prof. Andrzej Horzela (IFJ PAN) points out, "The Doppler phenomenon occurs in wave equations, which we say are local. We understand local here in that there is no delay between action and reaction. The principles of mechanics, for example, are local—a change in the resultant force acting on a body immediately results in a change in its acceleration.

"However, we all know that we can pick up a hot cup and before we feel it burning, a second or two passes. The phenomenon exhibits a certain delay; we say it is non-local, in other words, smeared in time. Do we therefore see the Doppler effect in the generalized telegraph equation describing time-smeared systems?"

Easy to ask, harder to answer. The trouble is in the math itself. If all we have in equations are derivatives and constants, then there is usually little trouble finding solutions. This is the case in the wave equation. The matter becomes more complicated when the equation contains only integrals, but even then one can often manage. Meanwhile, in the generalized telegraph equation, derivatives and integrals occur simultaneously.

At the heart of the Cracow physicists' paper was therefore the proof that solutions of the generalized telegraph equation can be constructed from much simpler to find solutions of the local equation. Here, a key role was played by the procedure known in stochastic process theory as subordination.

The following example helps us understand the concept of subordination. Imagine a man who has had too much to drink, but bravely tries to walk in a straight line. He takes one step and stands still, waiting for the world to stop spinning. He then takes another step, probably a little longer or shorter than the previous one, and stops again for an unspecified amount of time.

The mathematical description of such a movement, called random wandering, need not be trivial at all. What really matters, however, is not how much time our "wanderer" spends in a given place, but what distance he or she ultimately covers.

If the time between steps were equal, the description of the sailor's movement would be simpler and correspond to the movement of a sober person—it would simply be the sum of a sequence of successive, smoothly following steps.

"In our approach, subordination consists of replacing uniformly elapsing physical time, in which the equations are complicated, with a certain intrinsic time associated with physical time, which we do through an appropriate function containing information about the temporal non-locality of the process. This procedure simplifies the equations into a form that makes it possible to find their solutions," says co-author of the paper Tobiasz Pietrzak, M.Sc, a student at the Cracow Interdisciplinary Doctoral School.

Solutions of the ordinary telegraph equation show features typical of the Doppler effect. They show the presence of a clear, sharp frequency inflection, corresponding to the moment when the source passes the observer and there is an instantaneous, abrupt change in the pitch of the sound recorded by the observer.

Analogous behavior was observed by the Cracow physicists in the solutions of the generalized equation. It seems, therefore, that the Doppler effect is a fundamental feature of wave motion. However, that is not all. In the physical world, every wave has its wavefront, which, somewhat simplified, can be identified with its beginning and end. When we look at the front of the wave (and therefore its wavefront), the Doppler shift is easy to see.

It turns out that changes in wave frequency due to changes in the distance between the observer and the source also occur for waves that do not show the existence of a wavefront, e.g. defined over an unlimited area.

Research into the wave aspects of heat propagation may seem like a very abstract consideration, but its translation into everyday practice seems quite real. Physicists from the IPJ PAN point out that the knowledge they have gained can be used, in particular, in situations where the transport of heat over short distances is involved.

Examples include medical applications, where a better understanding of heat transport mechanisms may allow for the development of safer techniques for working with laser surgical instruments or finding a method to remove excess heat from burnt tissues more efficiently than before. Cosmetology, interested in minimizing unwanted thermal effects occurring during cosmetic procedures, may also benefit.

More information: T. Pietrzak et al, The generalized telegraph equation with moving harmonic source: Solvability using the integral decomposition technique and wave aspects, *International Journal of Heat and Mass Transfer* (2024). DOI: 10.1016/j.ijheatmasstransfer.2024.125373

Provided by Polish Academy of Sciences

Citation: From burns to the wave nature of heat—via the telegraph equation (2024, May 23) retrieved 16 June 2024 from https://phys.org/news/2024-05-nature-telegraph-equation.html