Vibrations of granular materials: Theoretical physicists shed light on an everyday scientific mystery

April 17 2024

Credit: CC0 Public Domain

Coffee beans in a jar and piles of rice or sand are examples of granular matter: materials composed of large numbers of macroscopic—rather than atomic scale—particles. Although granular matter is extremely familiar in everyday life, it represents an unexpected frontier in fundamental physics: Very little is understood about it.

In a new study published recently in the *European Physical Journal E*, Onuttom Narayan and Harsh Mathur, theoretical physicists at the University of California at Santa Cruz and Case Western Reserve University, respectively, shed light on the propagation of sound through granular materials particularly close to what is called "the jamming transition."

Understanding the properties of granular matter is important for many practical industrial applications. Remarkably, the problem of acoustic vibrations in granular matter has recently been a subject of discussion in pop culture: The newly released movie "Dune" has engendered a debate about whether sound can propagate through sand (it can).

The strangeness of granular matter can be seen by contemplating a pile of rice. If you push a pile of rice gently, it appears to be solid. But if you pick up some rice and let it slip through your hand, it pours like a liquid. Thus, a pile of rice is neither solid nor liquid. It is a granular material that has to be understood on its own terms.

To understand the jamming transition, imagine pouring coffee beans through a funnel with a narrow nozzle. If the beans are poured slowly, they will flow through the nozzle, but if a lot of beans are poured into the funnel rapidly, the flow will jam. The jamming transition occurs as the flow rate is increased: The material passes abruptly from a flowing state to a jammed one.

In the laboratory, researchers typically study packs of polystyrene beads which are more amenable to experimentation than coffee beans. It is found that such bead packs undergo acoustic vibrations at a set of characteristic frequencies. This set of characteristic frequencies is called the spectrum of the bead pack. The spectrum varies from bead pack to bead pack, so the problem is to develop a statistical understanding of the kinds of spectra that might arise.

Building on important prior work by many researchers, especially Yaroslav Beltukov (Ioffe Institute in Russia) and Giorgio Parisi (Sapienza University of Rome), Narayan and Mathur show there are certain statistical features of the spectra that are universal, while other features are non-universal. In this context, universal refers to features that would be shared by the vibrational frequencies of any sufficiently complex system; non-universal to features specific to jammed granular matter.

Narayan and Mathur show that the universal features of the spectra are described by random matrix theory, a branch of mathematics developed by nuclear physicists in the 1950s. The possibility that random matrix theory might be applicable to the vibrations of granular matter has important precursors. But in the new work, it is convincingly demonstrated for the first time that the spectra are described by a particular flavor of random matrix theory called the Laguerre ensemble.

Narayan and Mathur have also developed a model of the vibrations of jammed granular matter that is able to explain some of the non-universal features of the spectra. This model closely resembles a model developed by Narayan many years ago that was intended to solve a different important puzzle about granular matter: how stress is distributed in bead packs that are compressed.

Finding a unified description of different phenomena is a major goal of fundamental physics. An important goal for future work is to merge the two related models into a unified description of both stress distributions and vibrational spectra.

Granular matter is a reminder, Mathur and Narayan said, that one does not only have to look to the subatomic world or the universe on a cosmological scale to find important unsolved fundamental problems. Equally challenging and significant problems may be found in the everyday world around us.

More information: Onuttom Narayan et al, Vibrational spectrum of Granular packings with random matrices, *The European Physical Journal E* (2024). DOI: 10.1140/epje/s10189-024-00414-x

Provided by Case Western Reserve University