A mathematical bridge between the huge and the tiny

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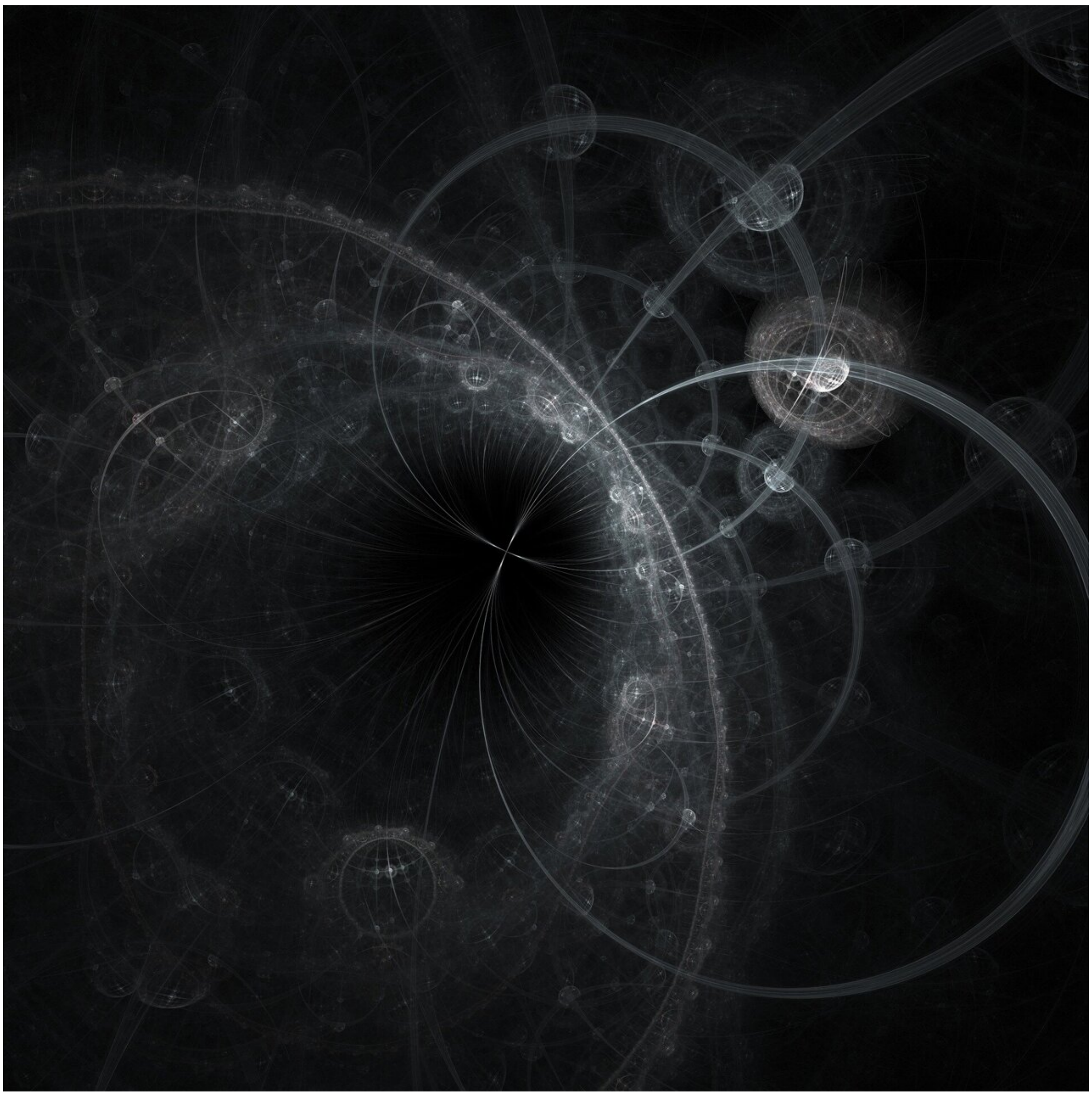

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A mathematical link between two key equations—one that deals with the very big and the other, the very small—has been developed by a young mathematician in China.

The mathematical discipline known as differential geometry is concerned with the geometry of smooth shapes and spaces. With roots going back to antiquity, the field flourished in the early 20th century, enabling Einstein to develop his general theory of relativity and other physicists to develop quantum field theory and the Standard Model of particle physics.

Gao Chen, a 29-year-old mathematician at the University of Science and Technology of China in Hefei, specializes in a branch known as complex differential geometry. Its complexity is not in dealing with complicated structures, but rather because it is based on complex numbers—a system of numbers that extends everyday numbers by including the square root of -1.

This area appeals to Chen because of its connections with other fields. "Complex differential geometry lies at the intersection of analysis, algebra, and mathematical physics," he says. "Many tools can be used to study this area."

Chen has now found a new link between two important equations in the field: the Kähler–Einstein equation, which describes how mass causes curvature in space–time in general relativity, and the Hermitian–Yang–Mills equation, which underpins the Standard Model of particle physics.

Chen was inspired by his Ph.D. supervisor Xiuxiong Chen of New York's Stony Brook University, to take on the problem. "Finding solutions to the Hermitian–Yang–Mills and the Kähler–Einstein equations are considered the most important advances in complex differential geometry in previous decades," says Gao Chen. "My results provide a connection between these two key results."

"The Kähler –Einstein equation describes very large things, as large as the universe, whereas the Hermitian–Yang–Mills equation describes tiny things, as small as quantum phenomena," explains Gao Chen. "I've built a bridge between these two equations." Gao Chen notes that other bridges existed previously, but that he has found a new one.

"This bridge provides a new key, a new tool for theoretical research in this field," Gao Chen adds. His paper describing this bridge was published in the journal *Inventiones mathematicae* in 2021.

In particular, the finding could find use in string theory—the leading contender of theories that researchers are developing in their quest to unite quantum physics and relativity. "The deformed Hermitian–Yang–Mills equation that I studied plays an important role in the study of string theory," notes Gao Chen.

Gao Chen now has his eyes set on other important problems, including one of the seven Millennium Prize Problems. These are considered the most challenging in the field by mathematicians and carry a $1 million prize for a correct solution. "In the future, I hope to tackle a generalization of the Kähler–Einstein equation," he says. "I also hope to work on other Millennium Prize problems, including the Hodge conjecture."

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