Paper offers a mathematical approach to modeling a random walker moving across a random landscape

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Credit: Qingbao Meng/Unsplash

Tiny particles like pollen grains move constantly, pushed and pulled by environmental forces. To study this motion, physicists use a "random walk" model—a system in which every step is determined by a random process. Random walks are useful for studying everything from tiny physics to diffusion to financial markets.

But what if the environment itself—and not just the walker—is random? "We can think of a town in which the elevation undulates in a random way, with the walker more likely to step downhill rather than uphill," says physicist and SFI Professor Sidney Redner.

A fundamental question in this scenario, he says, is to determine the time for the system to move from one arbitrary point to another. This quantity is called the "first-passage time," and researchers have solved it in one dimension, albeit using cumbersome calculations.

In a paper published in *Physical Review E*, Redner, together with SFI Program Postdoctoral Fellow James Holehouse, introduced a new way to efficiently determine all possible first-passage times and their probabilities. Their approach, which relies on heady math, captures the randomness of both the walker and the environment.

In the paper, they describe how to compute a "moment generating function"—a kind of mathematical machine for providing complete statistical information about the distribution of first-passage times.

Their approach could improve predictive analyses in a wide range of processes influenced by randomness, from changing biological populations to migration systems to the dynamics of financial instruments used to study markets. It builds on ideas that Redner first described in his 2001 book "A Guide to First Passage Processes" (and for which he's preparing a second edition.)

Researchers typically approach first-passage problems using enormous simulations, which start with initial systems and run through time to predict the time to reach a certain state. "But simulations are a really poor way to study [these systems]," Holehouse says.

Redner adds, "If you simulate some of these systems, you're guaranteed to get the wrong answer because you need to simulate so many instances of the system that to see the right answer would require a computation time that is beyond the age of the universe."

More information: James Holehouse et al, First passage on disordered intervals, *Physical Review E* (2024). DOI: 10.1103/PhysRevE.109.L032102. On *arXiv*: arxiv.org/abs/2307.08879

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