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# Validating the low-rank hypothesis in complex systems

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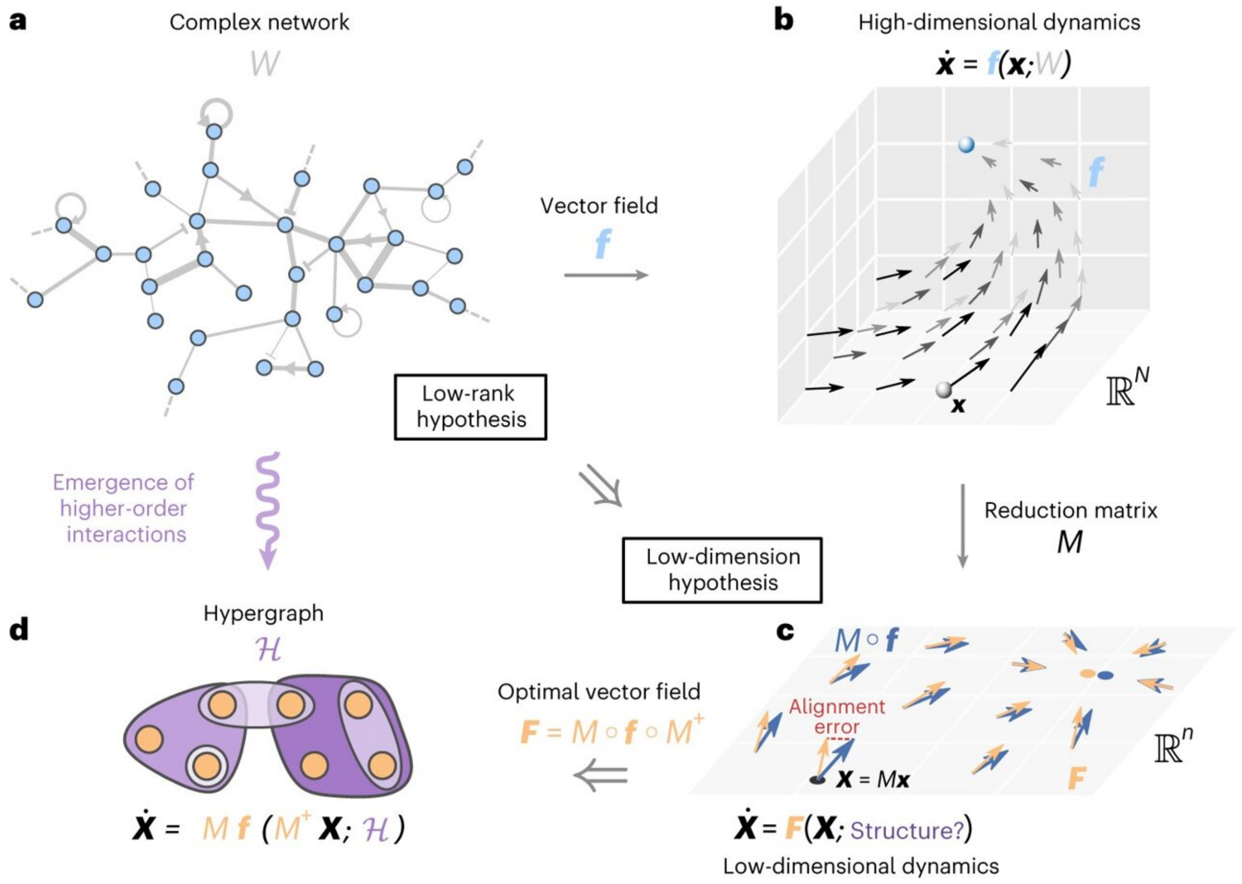

The low-rank hypothesis of complex systems and the emergence of higher-order interactions. Credit: *Nature Physics* (2024). DOI: 10.1038/s41567-023-02303-0

In a new study, scientists have investigated the pervasive low-rank hypothesis in complex systems, demonstrating that despite high-dimensional nonlinear dynamics, many real networks exhibit rapidly decreasing singular values, supporting the feasibility of effective dimension reduction for understanding and modeling complex system behaviors.

The findings of the study are published in *Nature Physics*.

Complex systems refer to intricate, interconnected structures or processes characterized by numerous components with nonlinear interactions, making their behavior challenging to predict from the properties of individual parts.

Examples include ecosystems, neural networks, and social structures, where collective interactions lead to emergent phenomena and self-organization. Understanding complex systems involves studying patterns, feedback loops, and dynamic behaviors across various scales, contributing to physics, biology, sociology, and network science.

Complex systems often pose challenges in understanding their large-scale behavior due to the high-dimensional nonlinear dynamics involved. Now, scientists led by Vincent Thibeault, a Ph.D. student at Université Laval in Québec, Canada, aim to address this challenge by exploring the intrinsic simplicity of complex systems and finding an optimal dimension for simplifying models.

"By reading a large spectrum of papers on the subject, from network science to neuroscience, Patrick and I came to a point where it was evident that there was a low-rank hypothesis made on the matrix used to describe real networks and the interactions in many high-dimensional nonlinear dynamical systems."

"With Antoine in our team, who has dedicated several years to advancing network science, we were confident to delve into this research," Thibeault told Phys.org.

## Low-rank hypothesis

The brain is a complex system with several interacting elements, which in this case are the neurons. Neurons communicate with each other through electrical signals known as action potentials.

When groups of neurons synchronize their firing, it can enhance the efficiency of information processing and transmission. This synchronized activity is an emergent phenomenon due to the collective phenomena of the parts and can alter their functions, leading to conditions like epilepsy.

"Despite this high dimensionality, the intricate network of interactions exhibits low effective dimensions. This implies that only a few well-chosen variables (or observables) can be sufficient to describe the emergent macroscopic properties of complex systems."

"Yet, one must be very careful when choosing the dimension to describe these systems, as one can lose the salient properties of the system and even create new types of interactions," explained Thibeault.

The researchers sought to validate this low-rank hypothesis, aiming to find an optimal dimension for dimensionality reduction. They wanted to understand if the dynamics of high-dimensional complex systems depend on the behavior of low-rank matrices and if this hypothesis holds for a wide range of networks.

## Singular value decomposition

The researchers employed a powerful mathematical tool to test their low-rank hypothesis, singular value decomposition (SVD). SVD is a technique from linear algebra that dissects a matrix into three essential components.

The left singular vectors (U) describe how components in the system relate to each other. The singular values (Σ) indicate the importance of each component, and the right singular vectors (V) capture how each component influences the overall system.

When applying SVD to the weight matrices of networks, the researchers focused on understanding the behavior of singular values. They observed a rapid decrease in these singular values when analyzing real networks, providing empirical evidence for the low-rank hypothesis.

This analysis allowed them to validate the low-rank hypothesis, confirming that the dynamics of high-dimensional complex systems can be effectively reduced to a lower dimension, providing insights into the optimal dimensionality for simplifying models and understanding emergent macroscopic properties.

In addition to validating the low-rank hypothesis through the rapid decrease of singular values, the researchers also found that this analysis enabled them to quantify the effective rank of networks.

Effective rank measurements, such as stable rank, provided quantitative indicators supporting the low-rank hypothesis. This further strengthened the understanding that despite the intricate and high-dimensional nature of complex systems, their behaviors can indeed be accurately captured with a significantly lower number of dimensions, offering a more manageable and insightful representation for scientific inquiry and modeling purposes.

"The origin of higher-order interactions was not even a subject that we thought about initially in our research process. In fact, after verifying the low-rank hypothesis, we were only concerned about finding an optimal dimension-reduction method," noted Thibeault.

## Experimental verification and adaptive systems

The researchers went one step further and ventured into the real-world complexity of networks.

Experimental scrutiny, including investigations into the connectome of Drosophila melanogaster, yielded empirical evidence by confirming the rapid decay of singular values.

A connectome is the complete map of the neural connections in the D. melanogaster, a species of fruit fly. This tangible verification transcends theoretical frameworks, affirming the applicability of the low-rank hypothesis in complex systems.

Thibeault highlighted the significance of these empirical insights, saying, "These abilities are vital in fields like ecology, epidemiology, and neuroscience, where making informed predictions and exerting some level of control are key objectives, even under strong simplifying assumptions."

"Identifying the limits of our mathematical models (like random graphs and dynamical systems) for describing natural phenomena is thus a fundamental task for the modeler, and establishing the ubiquity of the low-rank hypothesis is part of this effort for complex systems."

Looking ahead, the researchers envision an exploration of the origins of rapid singular value decreases in real networks, anticipating valuable insights into the resilience of complex adaptive systems.

Thibeault explained, "Complex systems are inherently adaptive systems, with the network of interactions and the system dynamics evolving according to its environment and inherent behavior."

"The models that describe such adaptation are much more intricate, making dimension reduction an essential tool to get insights into the system's functions and resilience. We plan to thoroughly investigate and discuss the implications of our observations on complex adaptive systems in the future.

Jianxi Gao has published a News & Views piece in the same journal issue on the work by Thibeault 's team.

**More information:** Vincent Thibeault et al, The low-rank hypothesis of complex systems, *Nature Physics* (2024). DOI: 10.1038/s41567-023-02303-0

Jianxi Gao, Intrinsic simplicity of complex systems, *Nature Physics* (2024). DOI: 10.1038/s41567-023-02268-0

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