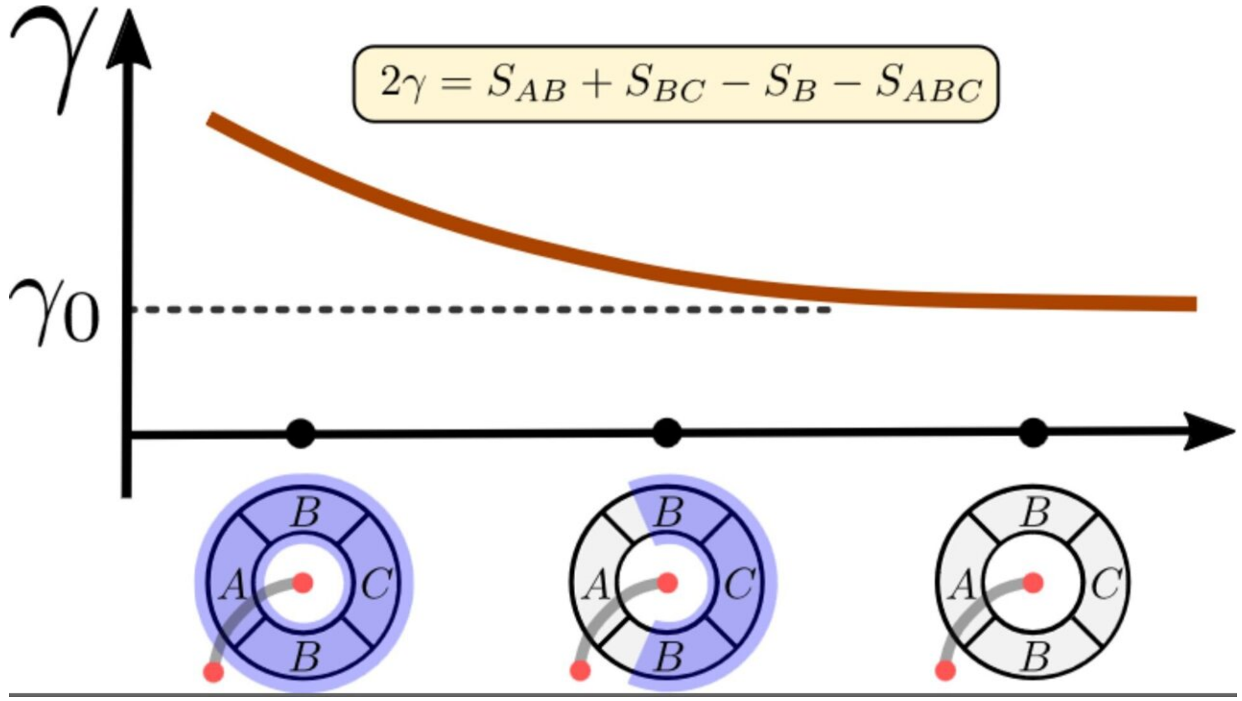
Scientists demonstrate the existence of a universal lower bound on topological entanglement entropy

October 27 2023, by Tejasri Gururaj



When the constant-depth circuit is removed, the value of topological entanglement entropy (γ) computed from the state returns to the ideal value (γ_0). Credit: Dr. Bowen Shi.

In a new study, scientists from the US and Taiwan have theoretically demonstrated the existence of a universal lower bound on topological entanglement entropy, which is always non-negative. The findings are published in the journal *Physical Review Letters*.

Quantum systems are bizarre and follow their own rules, with quantum states telling us everything we know about that system. Topological entanglement entropy (TEE) is a measure that provides insights into emergent non-local phenomena and entanglement in quantum systems with topological properties.

Given the fundamental role of quantum entanglement in quantum computing and various information applications, understanding TEE becomes essential for gaining insights into the behavior of quantum systems.

Extracting information from quantum systems

In quantum systems, it's often observed that the entanglement entropies follow an area law. This means that the entanglement between particles or regions is related to the area of the boundary that separates them. TEE is a specific term within the entanglement entropy that provides additional information. It's like a correction term that characterizes the topological phase of the system.

In condensed matter physics, a topological phase refers to a specific state of matter characterized by unique topological properties. These properties are associated with the behavior of particles within the material, such as anyons, and can be distinguished by their TEE values.

"TEE is a fascinating thing. By computing the entanglement entropy from a single ground state, we can learn the number of species of anyons (emergent particles that are neither boson nor fermion) of the phase of matter. It came out 18 years ago. I believe many people got inspiration from it. The research area I work on may not exist without these early works," Dr. Bowen Shi, lead author of the study, told Phys.org.

In many models, TEE is thought to have a universal value that characterizes the properties of the underlying topological phase. However, this is not always the case. TEE can differ between two states that are related by constant-depth circuits. These circuits are a specific type of quantum circuit operation that performs a series of quantum gates or transformations in a way that restricts their depth, meaning the number of sequential operations.

The key idea is that these circuits manipulate quantum states, and according to the theory, states related by such circuits should be in the same phase because the operations don't significantly alter the underlying physics.

However, this isn't always the case, and the variations in TEE between such states are often referred to as spurious TEE.

Dr. Shi underscores the transformative power of TEE, saying, "The first time I read the original TEE papers, I was in graduate school studying particle physics. Now, I study emergent particles, where certain properties naturally emerge with large degrees of freedom. My collaborators and I argued that we can now use a single wave function and the entanglement area law to predict the emergence of anyons and the correct TEE value."

Essentially, they have a tool for understanding and predicting the behavior of emergent particles and their entanglement characteristics.

TEE invariance and universal lower bound

The researchers wanted to understand the reliability of extracting universal properties from a ground-state wave function. To explore this, they focused on two-dimensional (2D) gapped ground states.

These states exist in 2D systems, such as thin films or 2D materials, and are characterized by an energy gap that separates the ground state from higher-energy excited states. This energy gap ensures the stability and well-defined nature of the ground state, making it an ideal platform for investigating TEE.

Following this, they introduced noise to the gapped ground states using a constant-depth circuit. This noise is akin to perturbations or disturbances in the system. They aimed to observe how the spurious TEE changed when the gapped ground state was perturbed. What they found was truly remarkable.

"We found that the new state must extract a larger value of TEE than the state without noise. In other words, the so-called spurious topological entanglement entropy is always non-negative," explained Dr. Shi.

This basically means there is a universal lower bound on TEE, which is consistently non-negative. In simple terms, the entanglement entropy within these 2D gapped ground states remains non-negative, regardless of the perturbations introduced by the constant-depth circuit.

Dr. Shi compared this to a glass being always lighter once we wipe away the dust on its surface. Wiping away dust from a glass doesn't make it heavier but rather reveals its true weight. Similarly, adding noise doesn't decrease the TEE but reveals an additional, non-negative TEE in the system.

Furthermore, the researchers made an important observation: TEE is invariant under constant-depth quantum circuits. This makes it a useful tool for understanding the underlying topological phase of the ground state.

The path forward

Speaking of the potential practical implications of their research, Dr. Shi said, "TEE computation is essential for identifying a material's underlying phase. Previous studies revealed that TEE formula failure in noisy states introduced uncertainty in results. Our lower bound reduces half of this uncertainty, offering practical value. With the rise of quantum computing and preparation of quantum states, our findings may also aid in these states."

The discovery of a universal lower bound on TEE, which is always non-negative, underscores the robustness of this entanglement measure even in the presence of perturbations introduced by constant-depth circuits.

There are still uncharted territories in this field. The researchers have laid the foundation for further investigations, such as exploring the generality of noise's impact on spurious TEE, specifically the role of constant-depth circuits, and delving into the behavior of TEE at finite temperatures.

These open questions promise exciting prospects for future research in the study of quantum systems.

More information: Isaac H. Kim et al, Universal Lower Bound on Topological Entanglement Entropy, *Physical Review Letters* (2023). DOI: 10.1103/PhysRevLett.131.166601

Citation: Scientists demonstrate the existence of a universal lower bound on topological entanglement entropy (2023, October 27) retrieved 28 April 2024 from https://phys.org/news/2023-10-scientists-universal-bound-topological-entanglement.html