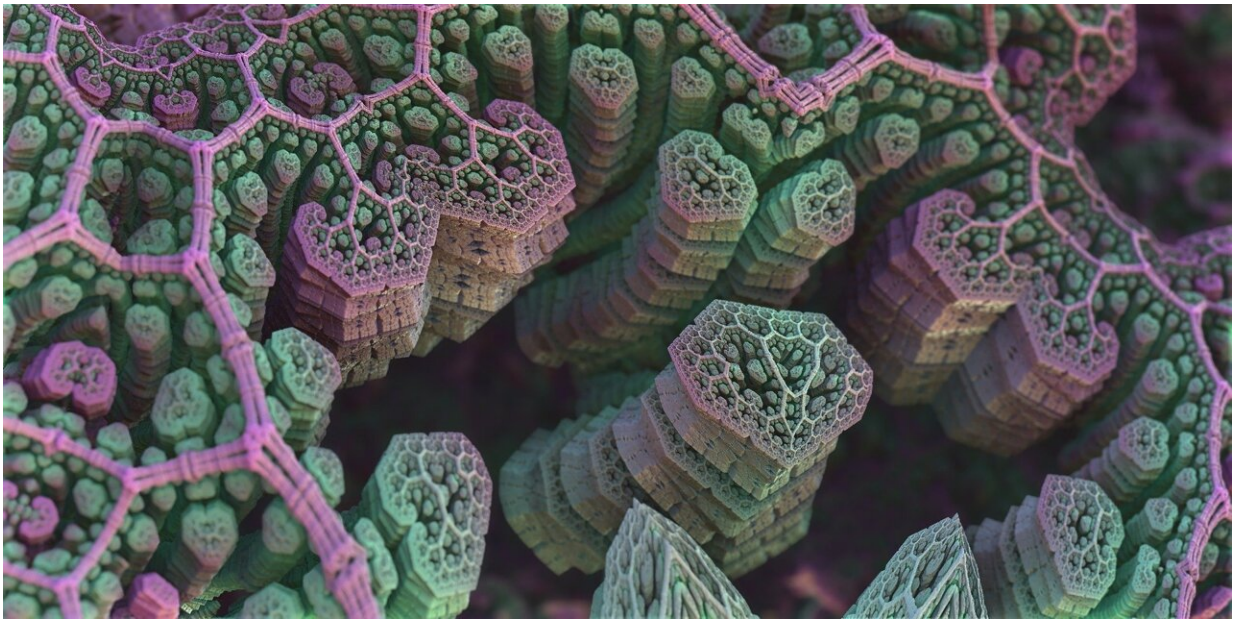


Mathematician develops equations that seek to bridge the micro and macro realms

February 7 2023



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Particles tend to behave differently once there are a lot of them. The microscopic models that describe their behavior quickly become very hard to calculate, because of the increasing complexity and the sheer volume of the data. This presents a challenge to researchers who are interested in the dynamics of macroscopic systems, such as nature, populations or tumors. By taking the number of particles to infinity, mathematician Jasper Hoeksma has developed a way to come up with

mathematical equations that work at both levels. He successfully defended his thesis at the department of Mathematics and Computer Science on Friday, February 3.

If you look at the world with a microscope, or at the universe with a telescope, it turns out that many things can be described in terms of [particles](#): particles that move, wiggle, hop or even pop in and out of existence. Particles can be molecules, bacteria, pixels, plants, planets, or sometimes even people.

In his thesis, mathematician Jasper Hoeksema has investigated the following question: what happens if you put many particles together, take a few steps back, and squint your eyes a bit?

"Instead of individual particles, you suddenly start to see vague colors or shades of gray. These roughly correspond to how many particles there are at any given spot. The more particles there are, and the further you step back, how sharper this picture becomes. Just like a painting in a museum that only makes sense when you look at it from a distance," explains Hoeksema.

"Mathematically, you can even talk about what happens if you take the number of particles to infinity, which is precisely what I did."

From micro to macro

Answering this question does more than merely satisfy a mathematical curiosity, says Hoeksema.

"It serves to bridge two ways of looking at reality that are often worlds apart. While microscopic models track the state of all particles in a system individually, macroscopic models look at the world from a more abstract level, describing qualities like movement, density, velocity and

temperature."

"Take the [behavior](#) of minute cracks in metal structures. It's relatively easy to describe the way these fissures attract and repel each other as a random (stochastic) process. But what if you want to explain how a metal beam bends? You can of course try to aggregate the insights you gained at the [microscopic level](#), and deduce from that how things work at a macroscopic level. But these calculations very quickly become very costly.

"As an alternative, engineers have come up with alternative models that, although less rigorous, can be used for experimental observations. My question is: how can I bring these models together? How can we derive a mathematical equation that fits both realities and is also rigorous?"

Two scenarios

For this, Hoeksema looked at two scenarios: one where particles attract and repel each other (like metal cracks), and one where particles are being added and removed (like changes in population, ecology and tumors).

In both cases he was able to show that if the number of particles approaches infinity, the density of the particles more and more satisfies a particular mathematical equation. Moreover, he calculated the probabilities that the particles differ from this expected behavior, and show that these probabilities become smaller and smaller.

"All of this guarantees that in the end we can throw away the positions of all the particles, and only have to worry about the mathematical equation itself," says Hoeksema. "This equation will allow you to make accurate predictions about the behavior of dynamic systems at a macroscopic level, without requiring massive amounts of computing power."

Hoeksema is very happy with the results of his mathematical investigations, which are part of a broader research project led by his supervisor Oliver Tse. "It is not often that mathematics succeeds in catching up with the practical research of physicists and their models. This shows why mathematics at a theoretical level matters, even if the applications are ten to twenty years away."

More information: Mean-field limits and beyond: Large deviations for singular interacting diffusions and variational convergence for population dynamics. pure.tue.nl/ws/portalfiles/portals/0203_Hoeksema_hf.pdf

Provided by Eindhoven University of Technology

Citation: Mathematician develops equations that seek to bridge the micro and macro realms (2023, February 7) retrieved 25 April 2024 from <https://phys.org/news/2023-02-mathematician-equations-bridge-micro-macro.html>

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