A model that predicts wrinkle patterns on the surface of toroidal structures as they expand or contract

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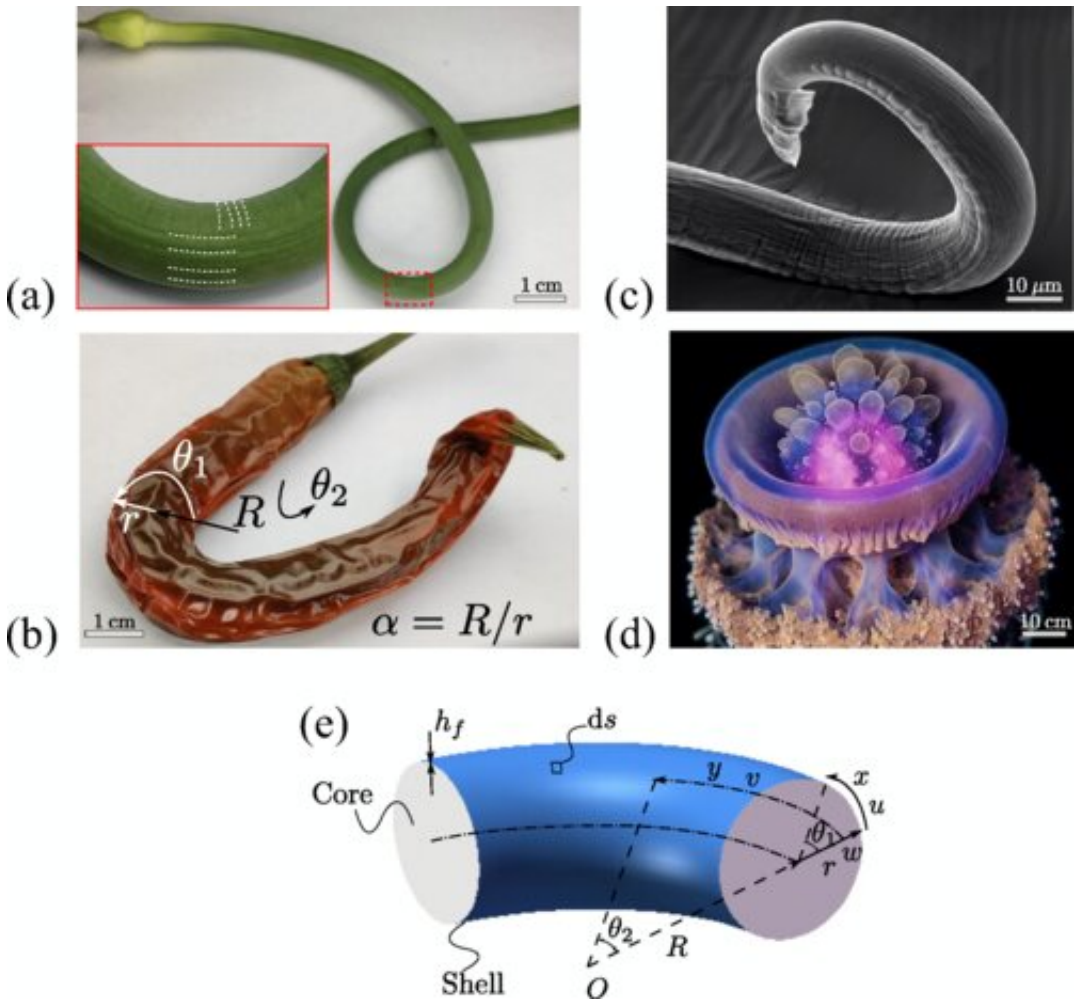

Surface wrinkling morphologies in various living matter with toroidal geometry. (a) Bidirectional stripes on a garlic. (b) Hexagonal-labyrinth bistable patterns on a dehydrated pepper. (c) Transverse and longitudinal wrinkles on an adult male Caenorhabditis elegans. Image obtained through SEM by Carolyn Marks and

David Hall of the Hall Lab in WormAtlas [20]. (d) Wrinkles in Crown jellyfish; photo by Jason Webb. (e) Geometry of a core-shell torus. Credit: *Physical Review Letters* (2023). DOI: 10.1103/PhysRevLett.130.048201

A team of researchers from Fudan University and Université de Lorraine has built a model that can predict the wrinkle patterns that will develop on toroidal structures if they expand or contract. In their paper published in the journal *Physical Review Letters*, the group describes using one type of mathematical model to develop another model that describes how growing or contracting impacts the surfaces of toroidal structures.

The researchers noticed similarities in wrinkle patterns found in natural biologically shaped toroidal structures, such as jellyfish, as they shrank. To create a model to predict such patterns they began with an already known math model used to describe toroidal structures in general. Such models, they note, are given parameters, one for the size of the hole and another to characterize stiffness.

The researchers then used the first model to create a second based on scaling laws that describe expansion and contraction of surface structures. They then tested the second model by generating general toroidal shapes that responded to environmental conditions such as heat or cold by shrinking or growing larger.

The researchers found that toroidal structural patterns differed depending on factors such as the size of the hole. They found, for example, that objects with larger holes tended to develop wrinkles closer to the hole than objects with smaller holes. They also found that objects with small holes tended to wrinkle away from the hole.

The researchers also found that the stiffness of the material making up

the toroidal structures had an impact on the shape of the patterns that made up the wrinkles that appeared as shrinking or expanding occurred. Structures with a moderately stiff skin, for example, tended to develop patterns with periodic hexagons.

The researchers found that their model could also be used with objects with somewhat different shapes. They found they could predict the wrinkling patterns that would emerge on a chili pepper, for example, as it dried. They further suggest that their model might prove useful to engineers working on designs for similarly shaped products to help predict how they might behave under various conditions.

More information: Ting Wang et al, Curvature-Regulated Multiphase Patterns in Tori, *Physical Review Letters* (2023). DOI: 10.1103/PhysRevLett.130.048201