Statistics star number crunches Christmas, shows how math can help your decorations look tree-mendous

December 6 2022

Professor Oliver Johnson has unravelled the numerical conundrums of Christmas ahead of the launch of his new book Numbercrunch. Credit: University of Bristol

The festive countdown is in full swing and numbers are uppermost in mind, as people manage tighter budgets and fill their social calendars with long-awaited gatherings now the worst of the COVID-19 pandemic

has passed.

Oliver Johnson, Professor of Information Theory at the University of Bristol, helped explain the constant stream of statistics during the pandemic. He has also been busy writing his debut book "Numbercrunch," out next year with Heligo Books, which reveals how numerical thinking can help resolve some of life's biggest conundrums.

To whet your appetite for his wizardry, Professor Johnson has turned his mathematical mindset to the equally challenging problem of number crunching Christmas.

From decorating your tree to stacking baubles, wrapping presents to picking out your favorite—or most disliked—chocolate, it turns out figures have an intriguing role to play. And they of course figure highly for Santa, who has endless lists and letters to get through, not to mention chimneys to squeeze down before the big day.

Professor Johnson said, "Even if you've grown out of advent calendars, it's impossible to escape the importance of numbers at Christmas. For instance, if you get the timings wrong on defrosting your turkey or miscount the number of places needed at the dinner table, it's likely to cause some serious festive frustration. But there are also some lesser-known and rather intriguing ways math can make your celebrations a little merrier."

So here are his top six numerical insights, shedding new light on the festive season and hopefully giving you some handy pointers on how to make them work to your advantage.

Trigonometree

Anyone who struggled to crack trigonometry at school is in luck because

this has nothing to do with the study of triangles and everything to do with randomness. Pay attention at the back and you might end up with a more attractive tree this Christmas.

Professor Johnson explained, "Randomness, as described in my book, is a tricky concept—humans aren't good at creating truly random patterns, partly because things that look random by eye often really aren't. This really matters when you decorate your Christmas tree.

"What you'd like is a nice spread of baubles, without too many of the same color next to one another. It seems natural to try decorating the tree 'at random,' but this won't lead to a good effect. Suppose you have 100 baubles and 100 branches: if you just put each bauble on a randomly chosen branch, then more than a third (about 37%) of the branches will have no decorations at all, whereas some might well have as many as four baubles.

"Similarly, there will be bare patches, just by random chance. In the same way, placing different colored baubles randomly will tend to lead to two or three baubles of the same color close together more often than we'd like. That means in fact, the best way to decorate your tree might be using a so-called quasi-random strategy, which lies somewhere between the very random and very structured extremes, and can be more pleasing on the eye."

All wrapped up

Watch out Martin Lewis, did you know Isaac Newton—who was even born on 25th December—can save you money on wrapping paper? As well as giving equations for gravity, his laws of motion and calculus could reap real rewards.

Professor Johnson said, "Newton's work in understanding the solutions

of certain equations was developed by Colin Maclaurin, a child prodigy who became professor of mathematics at the age of 19. This led to what is known as Maclaurin's inequality, which tells us that, among all the rectangular boxes with the same volume, the cube is the one with the smallest surface area.

"So, if you want to save on wrapping paper, you should look for presents that are close to cubes—in shape that is, not sugar cubes though they might go down well with Santa's reindeer. That's another reason not to forget the Chocolate Orange—it should cost you less to wrap than a thin flat bar with the same amount of chocolate inside.

The 12 Days of Pascal

No Christmas is complete without a rootin'-tootin' singalong of The Twelve Days of Christmas. Well done if you can remember all the lyrics but top marks are reserved for those who know the secret significance of the number of presents received each day.

Professor Johnson said, "The number of presents received each day and in total are hidden in the mathematical pattern known as Pascal's Triangle. On the first few days of Christmas, my true love sent me 1, then 1+2 = 3, then 1+2+3 = 6 presents. This sequence 1, 3, 6, 10, …, known as the triangular numbers, appears down one diagonal of the triangle.

"Similarly, the total number of presents I've received is 1, then 1+3 = 4, then 1+4+6 = 10. This sequence 1, 4, 10, 20, …, known as the tetrahedral numbers, can be seen down the next diagonal. This trick works because each entry of Pascal's Triangle is formed by taking the sum of the two numbers above it. For example, the 10 presents I receive on Day 4 are the same as the six presents I got the day before with the addition of four calling birds.

"By the same token, the 35 total presents I've received by Day 5 are made up as the 20 presents I'd received the day before, plus the 15 new presents arriving that day. That means, for those doing the math, by the twelfth day I will have received 364 presents in total. The presents I will have most of are the geese a-laying and swans a-swimming which first arrive on Days 6 and 7."

The Traveling Santa Problem

Children are rightly amazed by Santa Claus' incredible ability to travel fast enough to visit every house in the world in just one night. The vast distances and sheer volume of stops are so mind-boggling, they would stretch the most sophisticated supercomputer.

Professor Johnson said, "It is extremely hard to plan the most efficient route to visit a large collection of places—a challenge often referred to as the Traveling Salesman Problem. The largest case of this issue currently solved by humans had 85,900 places to visit, which took an extraordinary 136 years' worth of computing power.

"Even the best algorithms suffer from the fact that adding more destinations can lead to a rapid increase in computing time required. In practice, for logistics companies like Amazon planning their own Christmas deliveries, any efficiencies will be worth finding, but it may not be necessary to identify the very best delivery route.

"Given Santa's stellar track record, it might be reasonable to speculate whether, in addition to magical reindeer and a workforce of elves, he has access to a quantum computer, which could allow him to calculate his journey much more efficiently than the best methods currently available to humans."

A sweet ending

It's that time of year for the sinking feeling when you delve into the Christmas chocolate selection box and, as if by magic, pull out The One Which Nobody Likes. Last month brought the controversial news that Bounty chocolates have been excluded from a special edition of Celebrations, so that's one less choc to worry about, but how exactly do the odds stand against us?

Professor Johnson explained, "While it may be powerless to sweeten the pill, math can certainly help you understand what's going on. For example, what are the chances that the last chocolate left in the box is a nasty one? It's actually very simple: if our box has 24 nice chocolates and 6 nasty ones, there's a simple way to see the likelihood of the last one being nasty is 6/30, or 20%. That's the same chance that the first one is nasty, because you could imagine randomly pulling out all the chocolates and putting them in a long line—and then deciding which end of the line to start eating from.

"But what if people in your house don't follow the unwritten rules? What if everyone who pulls out a nasty chocolate returns it to the box with a certain probability, and has another go? We can mathematically analyze how the number of chocolates left of each type behave. This is an example of a Markov chain and, as also explained in my book, understanding these kinds of systems helps explain the behavior of queues in shops and call centers, the price of shares and the number of occupied hospital beds. These ideas even underpin the way Google search works.

"Put simply, the bigger the chance of people putting the bad chocolates back, the higher the chance that we are left with a nasty one at the end. For instance, with the same box as before, if half the time people put the bad chocolates back in the box then the chances of being left with one of

them will more than treble…to an unappetizing 64%. You have been warned!"

Full festive circle

Have you ever broken a Christmas bauble and blamed the way they were stored? Listen up for the best way to stack your balls.

Professor Johnson said, "Baubles are notoriously fragile and Christmas decorations have a nasty habit of taking up too much space. So what's the most efficient way of storing festive spherical objects, which also applies to the Chocolate Orange, walnuts, Brussels sprouts, and even snowballs?

"For nearly 400 hundred years, people had guessed the most efficient way to pack baubles was to use a regular hexagonal pattern in a layer, with each bauble touching six others around it, and with each layer being stacked offset above (with each bauble sitting in the gaps formed by the previous layers).

"However, this guess wasn't formally proved until 1998, using a computer search. In 2022, the Ukrainian mathematician Maryna Viazovska was awarded the Fields Medal (the Nobel Prize in math) for her work in proving the most efficient way to pack spheres in 8 or 24 dimensions. Although this is less likely to be practically useful for your Christmas, it could come in handy for your festive quiz or stashing away as impressive trivial knowledge."

While it won't be a stocking filler this year, Professor Johnson's book "Numbercrunch" is out in March and available for pre-order now.

Provided by University of Bristol