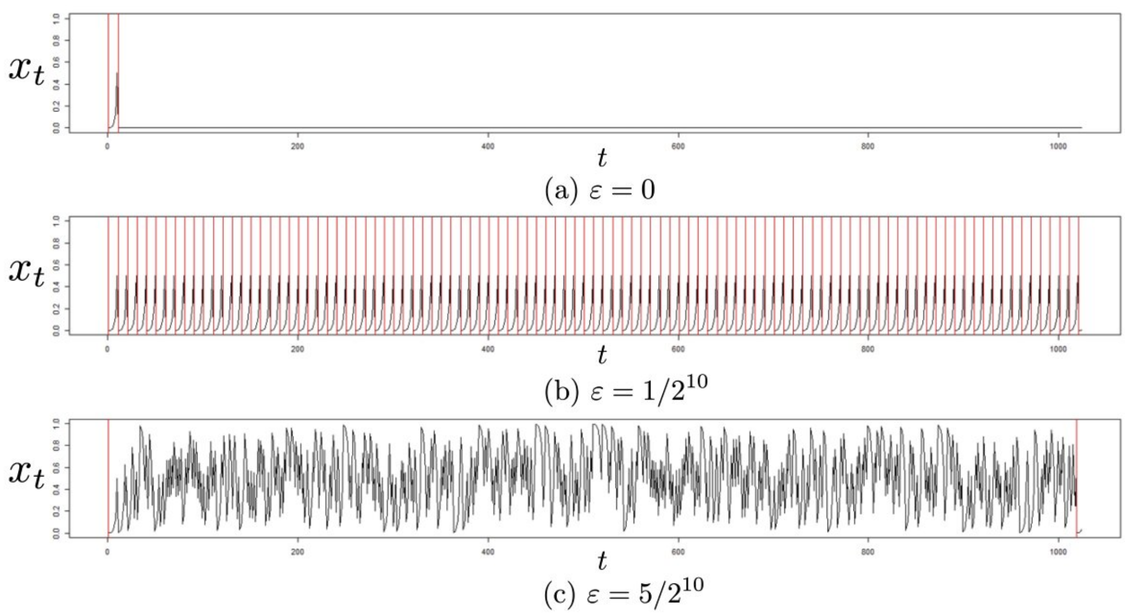
Perturbing the Bernoulli shift map in binary systems

November 3 2022



Bernoulli shift maps are theoretically chaotic, but produces converging outputs when implemented in binary systems. While this can be avoided by perturbing the map's state space, the parameter tuning for best perturbation has no theoretical basis. Now, researchers from Japan provide this theoretical support underlying the optimal parameter values. (a) If $\varepsilon = 0$ is implemented (no perturbation), the output converges to 0. (b) If the perturbation is not large enough (ε = 1/210), the output becomes periodic. (c) If ε is set to 5/210 using the proposed method in this study, the output does not converge. Credit: Prof. Tohru Ikeguchi from TUS and Dr. Noriyoshi Sukegawa from University of Tsukuba, Japan.

Is it possible for a deterministic system to be unpredictable? Although counter-intuitive, the answer is yes. Such systems are called "chaotic systems," which are characterized by sensitive dependence on initial conditions and long-term unpredictability. The behavior of such systems is often described using what is known as a "chaotic map." Chaotic maps finds applications in areas such as algorithm design, data analysis, and numerical simulations.

One well-known example of a chaotic map is the Bernoulli shift map. In practical applications of the Bernoulli shift map, the outputs are often required to have long periods. Strangely enough, however, when the Bernoulli shift map is implemented in a binary system, such as a digital computer, the output sequence is no longer chaotic and instead converges to zero.

To this end, perturbation methods are an effective strategy where a disturbance is applied to the state of the Bernoulli shift map to prevent its output from converging. However, the choice of parameters for obtaining suitable perturbations lacks a theoretical underpinning.

In a recent study in the journal *Chaos, Solitons & Fractals*, Professor Tohru Ikeguchi from the Tokyo University of Science in association with Dr. Noriyoshi Sukegawa from University of Tsukuba, both in Japan, have now addressed this issue, laying the theoretical foundations for effective parameter tuning.

"While numerical simulations can tell us which values of the parameters can prevent convergence, there is no theoretical background for choosing these values. In this paper, we aimed to investigate the theoretical support behind this choice," explains Prof. Ikeguchi.

Accordingly, the researchers made use of modular arithmetic to tune a dominant parameter in the perturbation method. In particular, they defined the best value for the parameter, which depended on the bit length specified in implementations. The team further analyzed the output period for which the parameter had the best value.

Their findings showed that the resulting periods came close to the trivial theoretical upper bounds. Based on this, the researchers obtained a complete list of the best parameter values for a successful implementation of the Bernoulli shift map.

Additionally, an interesting consequence of their investigation was its relation to Artin's conjecture on primitive roots, an open question in number theory. The researchers suggested that, provided Artin's conjecture were true, their approach would be theoretically guaranteed to be effective for any bit length.

Overall, the theoretical foundations put forth in this research are of paramount importance in the practical applications of chaotic maps in general. "A notable advantage of our approach is that it provides a theoretical support to the choice of best parameters. In addition, our analysis can also be partially applied to other chaotic maps, such as the tent map and the logistic map," highlights Dr. Sukegawa.

With distinct advantages, such as simplicity and ease of implementation, the Bernoulli shift maps is highly desirable in several practical applications. And, as this study shows, sometimes chaos is preferable to order.

More information: Noriyoshi Sukegawa et al, How to perturb Bernoulli shift map, *Chaos, Solitons & Fractals* (2022). DOI: 10.1016/j.chaos.2022.112793

Provided by Tokyo University of Science