Knots in the resonator: Elegant math in humble physics

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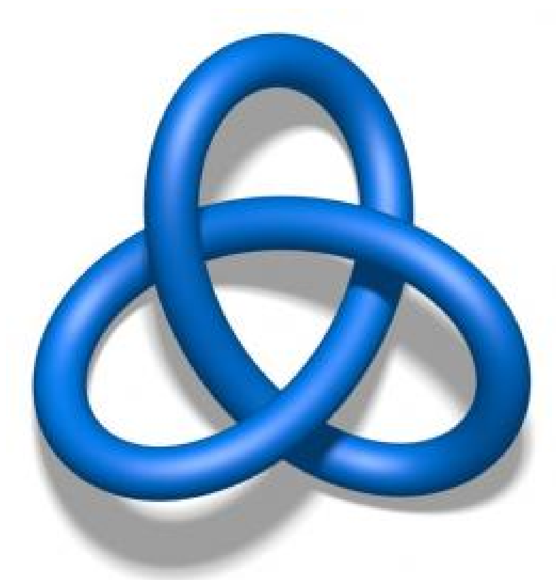

A trefoil knot. Credit: Wikipedia.

At the heart of every resonator—be it a cello, a gravitational wave detector, or the antenna in your cell phone—there is a beautiful bit of mathematics that has been heretofore unacknowledged.

Yale physicists Jack Harris and Nicholas Read know this because they started finding knots in their data.

In a new study in the journal *Nature*, Harris, Read, and their co-authors describe a previously unknown characteristic of resonators. A resonator is any object that vibrates only at a specific set of frequencies. They are ubiquitous in sensors, electronics, musical instruments, and other devices, where they are used to produce, amplify, or detect vibrations at specific frequencies.

The new characteristic the Yale team found results from equations that any high school algebra student would recognize, but which physicists had not appreciated as a basic principle of resonators.

It is this: If you make a graph of how the resonator's frequencies change while you "tune" the resonator—by varying its properties in almost any way—the graph will show braids and knots.

"The resonances twist around each other. It's awesome," Harris said. "It means any time you tune an instrument, you're making a braid. And if you tune it so that you keep two of the resonances equal, you're making a knot."

Harris is an experimental physicist. His bread and butter is exploring the ways topology and quantum mechanics influence sound and light. Often, he conducts experiments using resonators that trap light or sound in physical cavities.

Yet despite the high-tech nature of the work, there are analogs to working with much simpler instruments.

"If you're designing a violin and you want to know all the ways that it can vibrate, you're doing the same thing we are in my lab," Harris said. "It's the physics of vibration."

A few years ago, Harris was trying to understand some curious features that appeared in his data when he was tuning a cavity. He turned to his colleague Read, the Henry Ford II Professor of Physics and professor of applied physics and mathematics at Yale.

Read explained that these features were braids and were simply expressions of a fundamental math principle. "But when he explained that our data should contain trefoil knots, I was hooked," Harris said.

A trefoil knot is a figure found in the iconography of many cultures. It is also found in the artwork of M.C. Escher. Knots of this type are very familiar to mathematicians, but don't often crop up in physics.

Harris and Read designed an experiment in which they tuned three frequencies of a resonator and did, indeed, observe the predicted braids and knots.

The discovery, while basic for mathematics, may prove useful for physicists and engineers. "It's a potentially powerful tool, knowing that frequencies can braid in a resonator," Harris said. "That's because a braid is a topological object, meaning it doesn't change its essential character if you deform it a bit. It stays a braid unless you really mess it up. This is a special kind of robustness that we think can be used to prevent errors in applications that rely on precisely tuning resonators."

More information: Yogesh S. S. Patil et al, Measuring the knot of non-Hermitian degeneracies and non-commuting braids, *Nature* (2022). DOI: 10.1038/s41586-022-04796-w

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