The Riemann conjecture unveiled by physics

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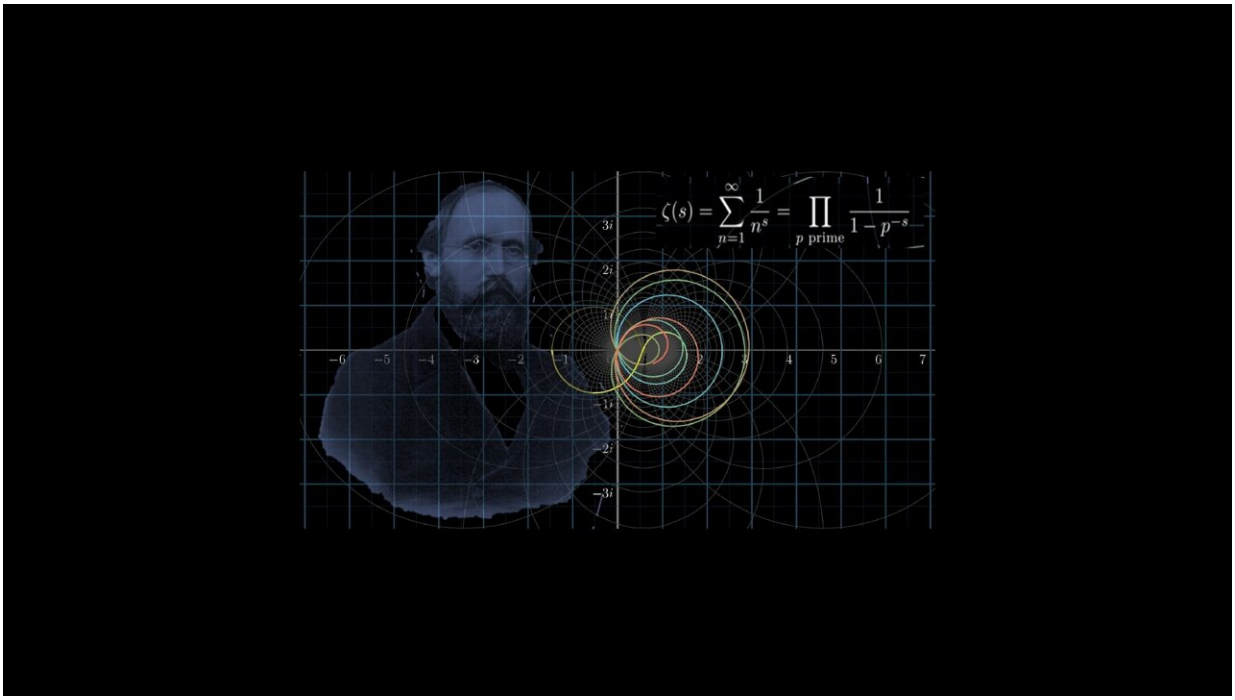

In the article, Giuseppe Mussardo and Andrè Leclair showed that there is instead an elegant explanation of the alignment of zeros along the ½ axis of the Riemann function (as well as of infinite similar functions, the so-called Dirichlet functions), ultimately due to a totally unexpected reason: the presence of a chaotic motion and the laws of probability that rule it. As a matter of fact, Mussardo and Leclair proved the existence of a Brownian motion, hidden behind all these infinite functions. Credit: Giuseppe Mussardo

A mystery of mathematics that has remained unsolved for more than 150 years can be unraveled thanks to a completely unexpected approach

coming from statistical physics. This is the important conclusion of Giuseppe Mussardo, professor of Theoretical Physics at SISSA, and Andrè Leclair of Cornell University reported in an article just published in the *Journal of Statistical Mechanics* (*JSTAT*). The two scientists have shown that not only one can arrive at the solution to one of the most famous problems in mathematics, the Riemann conjecture, but that it is the physics of chaotic motions and the probability laws that regulate them that provide the elegant key to understand this great mathematical enigma. The research behind the article just published, lasted three years and the final part of it, the authors said, was "a real tour de force in the data analysis of an incredibly large set of prime numbers, the basic constituents of arithmetic, i.e. the real atoms of mathematics."

Infinite zeros along a vertical line: an enigma that has lasted since 1859

The fact that mathematics provides physics with the right language to formulate the laws of nature is in the logic of things. The prospect that physics provides the key to understanding a genuine mystery of mathematics is, conversely, a rather unusual and extraordinary fact. This is the case of the Riemann conjecture, one of the most famous problems in mathematics. In 1859 the German mathematician Bernhard Riemann presented at the Berlin Academy of Sciences an article destined to change the history of mathematics. It concerned the mystery of prime numbers and the possibility of predicting their elusive distribution with astonishing accuracy. "At the heart of Riemann's argument there was a conjecture, which he was not able to prove, about the location of an infinite number of zeros in the complex plane of a particular function, known as the Riemann function. These zeros seem to magically align themselves along a vertical line with an abscissa exactly equal to ½ and until now no one has ever been able to understand the reason for such an incredible regularity," explains Giuseppe Mussardo. In a recent article

published in the Journal of Statistical Mechanics (JSTAT), Giuseppe Mussardo and Andrè Leclair showed that there is instead an extremely elegant explanation of the alignment of zeros along the ½ axis of the Riemann function (as well as of infinite similar functions, the so-called Dirichlet functions), ultimately due to a totally unexpected reason: the presence of a chaotic motion and the laws of probability that rule it. As a matter of fact, Mussardo and Leclair proved the existence of a Brownian motion, hidden behind all these infinite functions.

The Brownian motion behind the Riemann conjecture

The Brownian motion, a key phenomenon in statistical mechanics, understood for the first time by Albert Einstein in 1906, is the chaotic and disordered motion of the atoms of a gas due to the very high frequency of their collisions. In the Brownian motion, ½ is the universal exponent which rules how atoms spread as the time goes by, an incredibly robust exponent due to the probabilistic laws discovered by the great Gauss and entering his famous central limit theorem. "Our hypothesis on the Brownian nature of the Riemann conjecture, supported by a series of probabilistic results that we proved in Number Theory, has been accompanied by a massive and extremely precise statistical analysis done along the infinite sequence of prime numbers, a real tour de force which kept us busy for about three years," explains Giuseppe Mussardo. "The fact that the explanation of Riemann's conjecture comes from physics, i.e. from statistical mechanics and the surprising connections of this field with a genuinely mathematical topic such as number theory, reveals at once the great unity of scientific knowledge and, at the same time, increases our astonishment of facing such a profound fact," is the final comment of the two authors.

The research was published in the *Journal of Statistical Mechanics: Theory and Experiment*.

More information: Giuseppe Mussardo et al, Randomness of Möbius coefficients and Brownian motion: growth of the Mertens function and the Riemann hypothesis, *Journal of Statistical Mechanics: Theory and Experiment* (2021). DOI: 10.1088/1742-5468/ac22fb

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