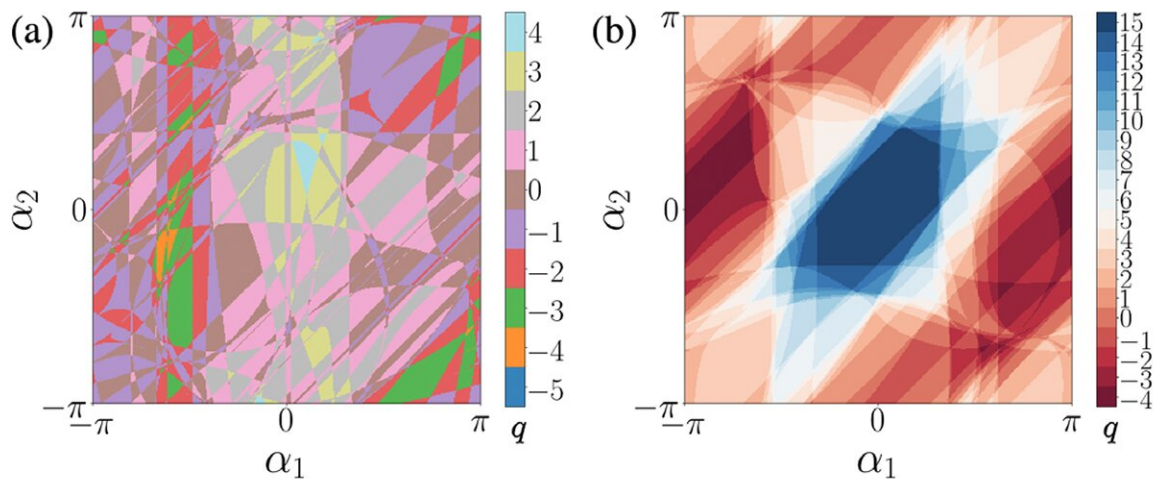


When is a 'basin of attraction' like an octopus?

November 4 2021



Credit: Santa Fe Institute

Mathematicians who study dynamical systems often focus on the rules of attraction. Namely, how does the choice of the starting point affect where a system ends up? Some systems are easier to describe than others. A swinging pendulum, for example, will always land at the lowest point no matter where it starts.

In [dynamical systems](#) research, a "[basin](#) of attraction" is the set of all the starting points—usually close to one another—that arrive at the same final state as the system evolves through time. For straightforward systems like a swinging pendulum, the shape and size of a basin is comprehensible. Not so for more complicated systems: those with dimensions that reach into the tens or hundreds or higher can have wild geometries with fractal boundaries.

In fact, they may look like the tentacles of an octopus, according to new work by Yuanzhao Zhang, physicist and SFI Schmidt Science Fellow, and Steven Strogatz, a mathematician and writer at Cornell University. The convoluted geometries of these high-dimensional basins can't be easily visualized, but in a new paper published in *Physical Review Letters*, the researchers describe a simple argument showing why basins in systems with multiple attractors should look like high-dimensional octopi. They make their argument by analyzing a simple model—a ring of oscillators that, despite only interacting locally, can produce myriad collective states such as in-phase synchronization. A high number of coupled oscillators will have many attractors, and therefore many basins.

"When you have a high-dimensional system, the tentacles dominate the basin size," says Zhang.

Importantly, the new work shows that the volume of a high-dimensional basin can't be correctly approximated by a hypercube, as tempting as it is. That's because the hypercube fails to encompass the vast majority—more than 99%—of the points in the basin, which are strung out on tentacles.

The paper also suggests that the topic of high-dimensional basins is rife with potential for new exploration. "The geometry is very far from anything we know," says Strogatz. "This is not so much about what we found as to remind people that so much is waiting to be found. This is

the early age of exploration for basins."

The work may also have real-world implications. Zhang points to the [power grid](#) as an example of important high-dimensional systems with multiple basins of attraction. Understanding which starting points lead to which outcomes may help engineers figure out how to keep the lights on.

"Depending on how you start your grid, it will either evolve to a normal operating state or a disruptive state—like a blackout," Zhang says.

More information: Yuanzhao Zhang et al, Basins with Tentacles, *Physical Review Letters* (2021). [DOI: 10.1103/PhysRevLett.127.194101](https://doi.org/10.1103/PhysRevLett.127.194101)

Provided by Santa Fe Institute

Citation: When is a 'basin of attraction' like an octopus? (2021, November 4) retrieved 27 April 2024 from <https://phys.org/news/2021-11-basin-octopus.html>

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