# Anomalous quantum transport phenomena observed in fractal photonic lattices

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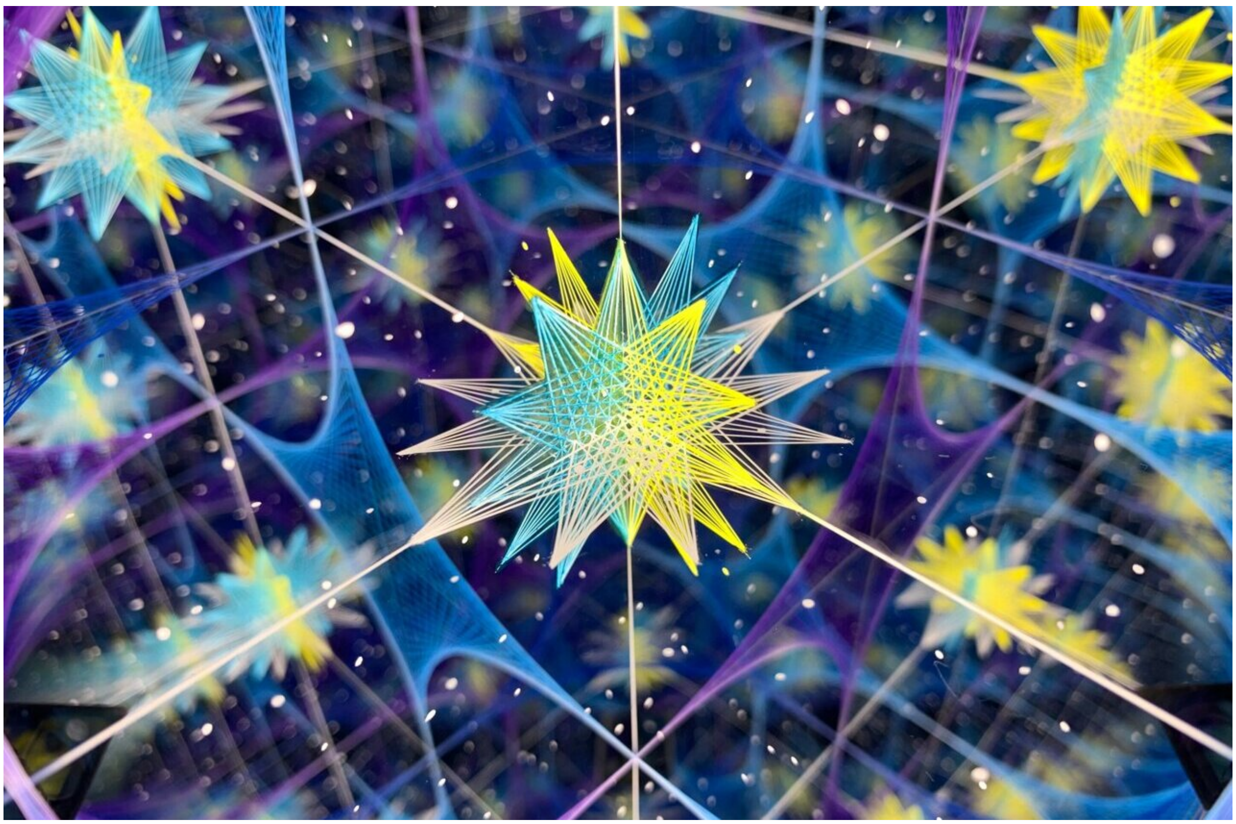

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Fractals are complex structures that usually exhibit self-similarity and have a non-integer dimension. The terminology "fractal" was first introduced by the famous mathematician Benoit B. Mandelbrot. He noticed that here and there, many natural objects were fractals, such as snowflakes, branching trees, coastline, etc. Outside of nature, fractal patterns or structures are also artificially created. One famous fractal type, Sierpinski gaskets, are widely used not only in the decoration of churches in ancient times, but also in modern artificial device engineering. So far, the feature of fractality has been reported in a wide range of fields including quantum mechanics, optics, finance, physiology, etc.

The aesthetic appeal of the appearance of fractals derives from the property of self-similarity. Physicists are also interested in the subtle physical law embedded in these unconventional systems, which are non-integer dimensional. Euclidean geometry is of integer dimension, and physical laws are mostly introduced in the case of integer-dimensional space. However, anomalous phenomena might occur in a different situation. Though there have been abundant theoretical and numerical studies in recent decades, experimental investigations of quantum transport in fractal space remain elusive.

Recently, a research group led by Prof. Xian-Min Jin from Shanghai Jiao Tong University, in collaboration with Prof. C. Morais Smith from Utrecht University, have experimentally investigated quantum transport dynamics in fractal space and observed anomalous phenomena. By using femtosecond laser direct writing techniques, the researchers were able to fabricate photonic lattices whose profile is fractal. Three typical types of fractals, Sierpinski gaskets, Sierpinski carpets and dual Sierpinski carpets, were precisely mapped to the photonic lattices. They are different either in Hausdorff dimension (i.e., the fractal dimension) or in geometry. Although dual Sierpinski carpets inherit the Hausdorff dimension of Sierpinski carpets, they have completely different geometry. The differences among the three fractals enable the researchers to investigate the interplay between quantum transport and fractality.

In the research, quantum walk, the quantum analog to the classical random walk, was used as a model to investigate quantum transport. Photons were launched into the photonic lattices to perform continuous-time quantum walks. The length of the lattices determines the evolution time of photons. By writing photonic lattices with incremental length, the researchers managed to capture the evolution results of photons at different moments and thus unveiled the quantum transport dynamics. The mean square displacement (MSD) was applied to characterize the quantum transport dynamics.

The results show that the transport dynamics can hardly be described by a single regime. It usually undergoes several stages, such as the normal regime, the fractal regime and the final saturation, which is different from the regular case. It is worth stressing that in contrast to translational-invariant lattices where the MSD scales quadratically, the MSD (in the fractal regime) is solely determined by the Hausdorff dimension. This anomalous phenomenon coincides well with the theoretical proposal of Fleischmann et al. The researchers have also further confirmed the robustness of the proposed relation by performing their simulation in a considerable large fractional space, and by investigating the independence of the relation on the input site (i.e., the position where the photons are launched into the lattices).

The research paves the way to a deeper understanding of physical law in fractional space. In addition to the fundamental interest in physics, it might shed some light on whether quantum mechanics plays any role in the transport in biological systems such as fractal-like brain hierarchy and branching trees where energy transport or information transport happens all the time. From the aspect of quantum algorithm, the realization of fractal photonic lattices lays a foundation for the experimental exploration of quantum spatial search based on continuous-time quantum walk.

**More information:** Mandelbrot, B. B. Fractals: Form, Chance and Dimension (W. H. Freeman, 1977)

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