Mathematicians find core mechanism to calculate tipping points

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At a tipping point, the system state may change slowly or abruptly. Credit: Emiliano Arano / Pexels

Climate change, a pandemic or the coordinated activity of neurons in the brain: In all of these examples, a transition takes place at a certain point from the base state to a new state. Researchers at the Technical University of Munich (TUM) have discovered a universal mathematical structure at these so-called tipping points. It creates the basis for a better understanding of the behavior of networked systems.

It is an essential question for scientists in every field: How can we predict and influence changes in a networked system? "In biology, one example is the modeling of coordinated neuron activity," says Christian Kühn, professor of multiscale and stochastic dynamics at TUM. Models of this kind are also used in other disciplines, for example when studying the spread of diseases or climate change.

All critical changes in networked systems have one thing in common: a tipping point where the system makes a transition from a base state to a new state. This may be a smooth shift, where the system can easily return to the base state. Or it can be a sharp, difficult-to-reverse transition where the system state can change abruptly or "explosively." Transitions of this kind also occur in climate change, for example with the melting of the polar ice caps. In many cases, the transitions result from the variation of a single parameter, such as the rise in concentrations of greenhouse gases behind climate change.

Similar structures in many models

In some cases—such as climate change—a sharp tipping point would have extremely negative effects, while in others it would be desirable. Consequently, researchers have used mathematical models to investigate how the type of transition is influenced by the introduction of new parameters or conditions. "For example, you could vary another parameter, perhaps related to how people change their behavior in a pandemic. Or you might adjust an input in a neural system," says Kühn. "In these examples and many other cases, we have seen that we can go from a continuous to a discontinuous transition or vice versa."

Kühn and Dr. Christian Bick of Vrije Universiteit Amsterdam studied existing models from various disciplines that were created to understand certain systems. "We found it remarkable that so many mathematical structures related to the tipping point looked very similar in those models," says Bick. "By reducing the problem to the most basic possible equation, we were able to identify a universal mechanism that decides on the type of tipping point and is valid for the greatest possible number of models."

Universal mathematical tool

The scientists have thus described a new core mechanism that makes it possible to calculate whether a networked system will have a continuous or discontinuous transition. "We provide a mathematical tool that can be applied universally—in other words, in theoretical physics, the climate sciences and in neurobiology and other disciplines—and works independently of the specific case at hand," says Kühn.

More information: Christian Kuehn et al, A universal route to explosive phenomena, *Science Advances* (2021). DOI: 10.1126/sciadv.abe3824

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