Researchers derive urban scaling laws from the 3D geometry of a city

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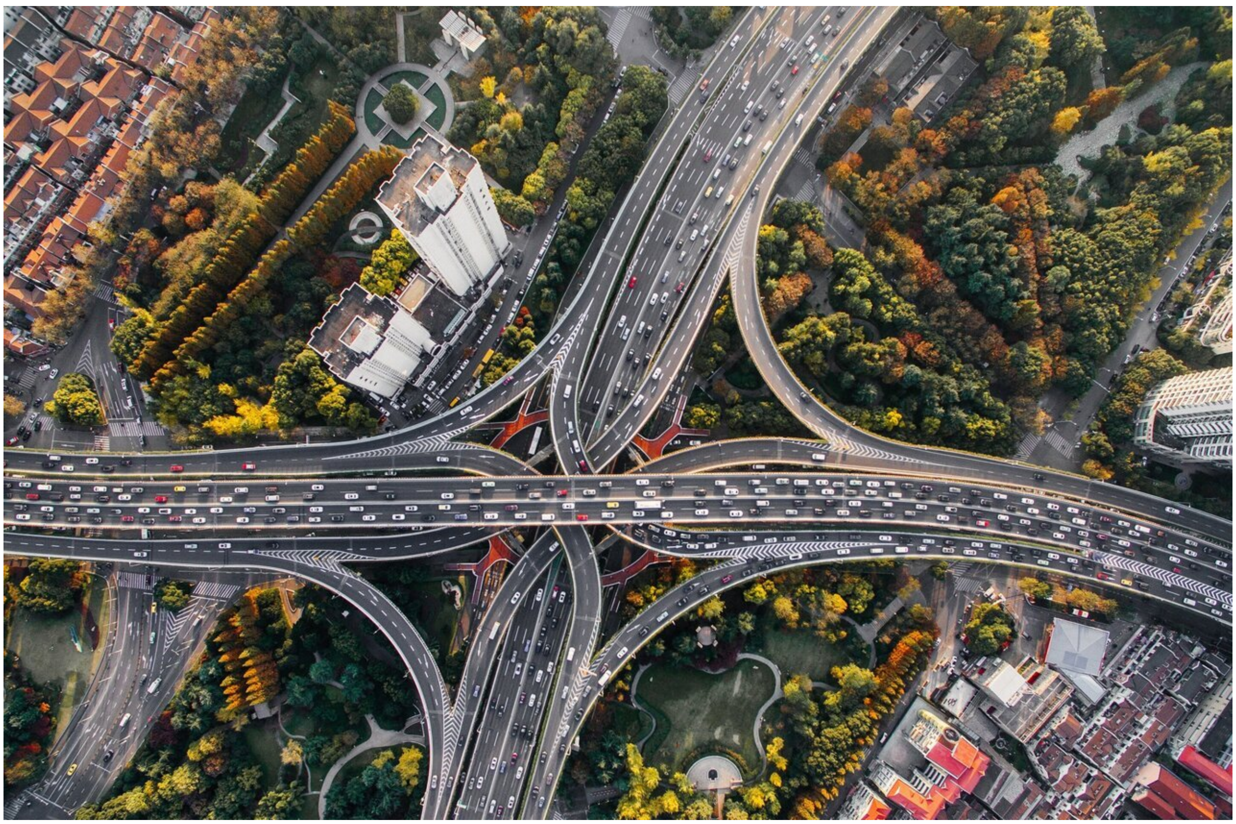

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When complex systems double in size, many of their parts do not. Characteristically, some aspects will grow by only about 80 percent, others by about 120 percent. The astonishing uniformity of these two growth rates is known as "scaling laws." Scaling laws are observed everywhere in the world, from biology to physical systems. They also apply to cities. Yet, while a multitude of examples show their presence, reasons for their emergence are still a matter of debate.

A new publication in the *Journal of The Royal Society Interface* now provides a simple explanation for urban scaling laws: Carlos Molinero and Stefan Thurner of the *Complexity Science Hub Vienna* (CSH) derive them from the geometry of a city.

Scaling laws in cities

An example of an urban scaling law is the number of gas stations: If a city with 20 gas stations doubles its population, the number of gas stations does not increase to 40, but only to 36. This growth rate of about 0.80 per doubling applies to much of the infrastructure of a city. For example, the energy consumption per person or the land coverage of a town rises by only 80 percent with each doubling. Since this growth is slower than what is expected from doubling, it is called sub-linear growth.

On the other hand, cities show more-than-doubling rates in more socially driven contexts. People in larger cities earn consistently more money for the same work, make more phone calls, and even walk faster than people in smaller towns. This super-linear growth rate is around 120 percent for every doubling.

Remarkably, these two growth rates, 0.8 and 1.2., are showing up over and over again in literally dozens of city-related contexts and applications. However, so far it is not really understood where these numbers come from.

It's all in the geometry

Stefan Thurner and former CSH researcher Carlos Molinero, who worked on this publication during his time in Vienna, now show that these scaling laws can be explained by the spatial geometry of cities. "Cities are always built in a way that infrastructure and people meet," says Molinero, an urban science expert. "We therefore think that scaling laws must somehow emerge from the interplay between the places people live in, and the spaces they use to move through a city—basically its streets."

"The innovative finding of this paper is how the spatial dimensions of a city relate to each other," adds complexity researcher and physicist Stefan Thurner.

Fractal geometry

To come to this conclusion, the researchers first mapped three-dimensionally where people live. They used open data for the height of buildings in more than 4,700 cities in Europe. "We know most of the buildings in 3D, so we can estimate how many floors a building has and how many people live in it," says Thurner. The scientists assigned a dot to every person living in a building. Together, these dots form sort of a "human cloud" wthin a city.

Clouds are fractals. Fractals are self-similar, meaning that if you zoom in, their parts look very similar to the whole. Using the human cloud, the researchers were able to determine the fractal dimension of a city's population: They retrieved a number that describes the human cloud in every city. Similarly, they calculated the fractal dimension of cities' road networks.

"Although these two numbers vary widely from city to city, we discovered that the ratio between the two is a constant," Thurner says. The researchers identified this constant as the "sublinear scaling exponent."

Aside from the elegance of the explanation, the finding has potential practical value, as the scientists point out. "At first sight this looks like magic, but it makes perfect sense if one takes a closer look," Thurner says. "It's this scaling exponent that determines how the properties of a city change with its size, and that is relevant because many cities around the world are growing rapidly."

A formula for sustainable urban planning

The number of people living in cities worldwide is expected to roughly double in the next 50 to 80 years. "Scaling laws show us what this doubling means in terms of wages, crime, inventiveness or resources needed per person—all this is important information for urban planners," Thurner points out.

To know the scaling exponent of a particular city could help urban planners to keep the gigantic resource demands of urban growth at bay. "We can now think specifically about how to get this number as small as possible, for example through clever architectural solutions and radically different approaches to mobility and infrastructure construction," Stefan Thurner is convinced. "The smaller the scaling exponent, the higher the resource efficiency of a city," he concludes.

More information: How the geometry of cities determines urban scaling laws, *Journal of The Royal Society Interface* (2021).
rsif.royalsocietypublishing.or … .1098/rsif.2020.0705

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