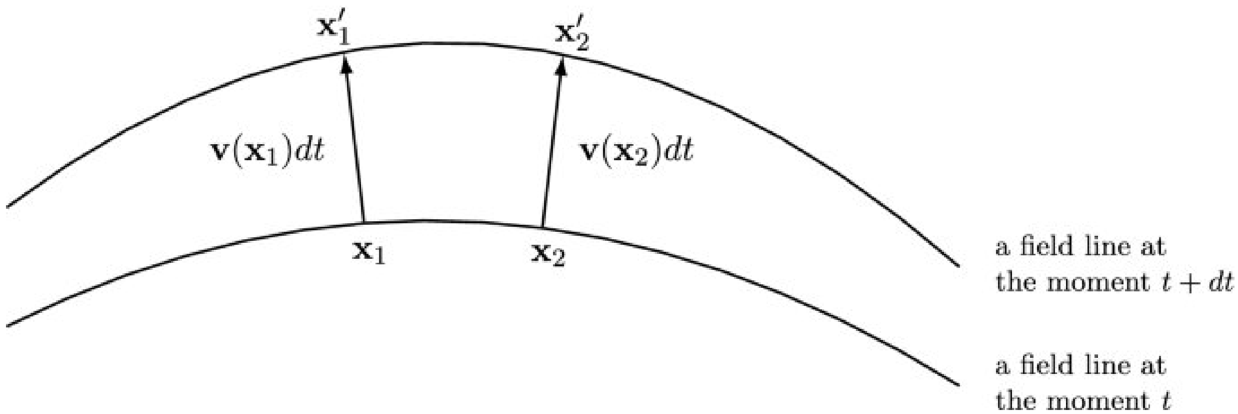
Tracking the evolution of Maxwell knots

January 18 2021, by Robert Lea



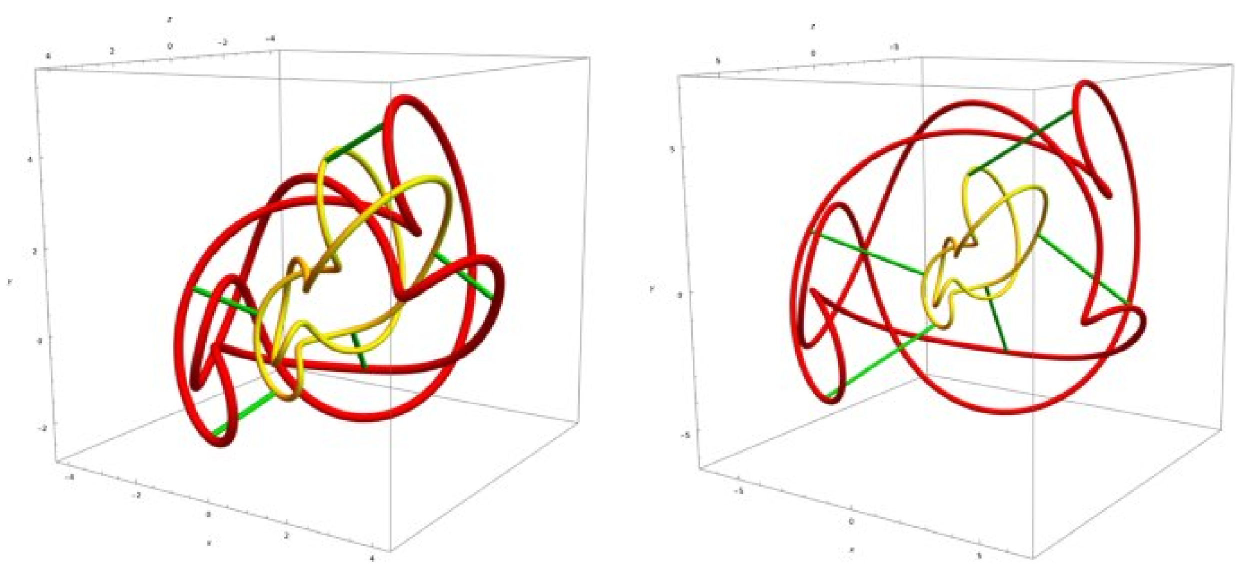
The picture demonstrates the condition that the vector field v should obey to define self-consistent time evolution of the field lines. For any two points on a field line at the moment t the ends of the vectors vdt at the corresponding points lie on a field line that is defined at the moment t+dt DOI 10.1140/epjc/s10052-020-08745-7

Maxwell equations govern the evolution of electromagnetic fields with light being a particular solution of these equations in spaces devoid of electric charge. A new study published in *EPJ C* by Alexi Morozov and Nikita Tselousov, from the Moscow Institute of Physics and Technology and the Institute of Transmission Problems, Russia, respectively, details peculiar solutions to the Maxwell equations—so-called Maxwell knots. The research could have applications in the fields of mathematical physics and string theory.

"We usually think of light as the plane waves. It was a breakthrough when 'knotted' light solutions were discovered," explains Tselousov. "The knot nature of these solution consists in the structure of the electric and magnetic field lines. One can observe that some of the field lines are closed loops and non-trivially knotted."

Electric and magnetic field lines change over time obeying the Maxwell equations. As the fields change their field lines somehow move in the space. Whilst researchers can't track an arbitrary field line, closed field lines are special and can be observed as they evolve over time.

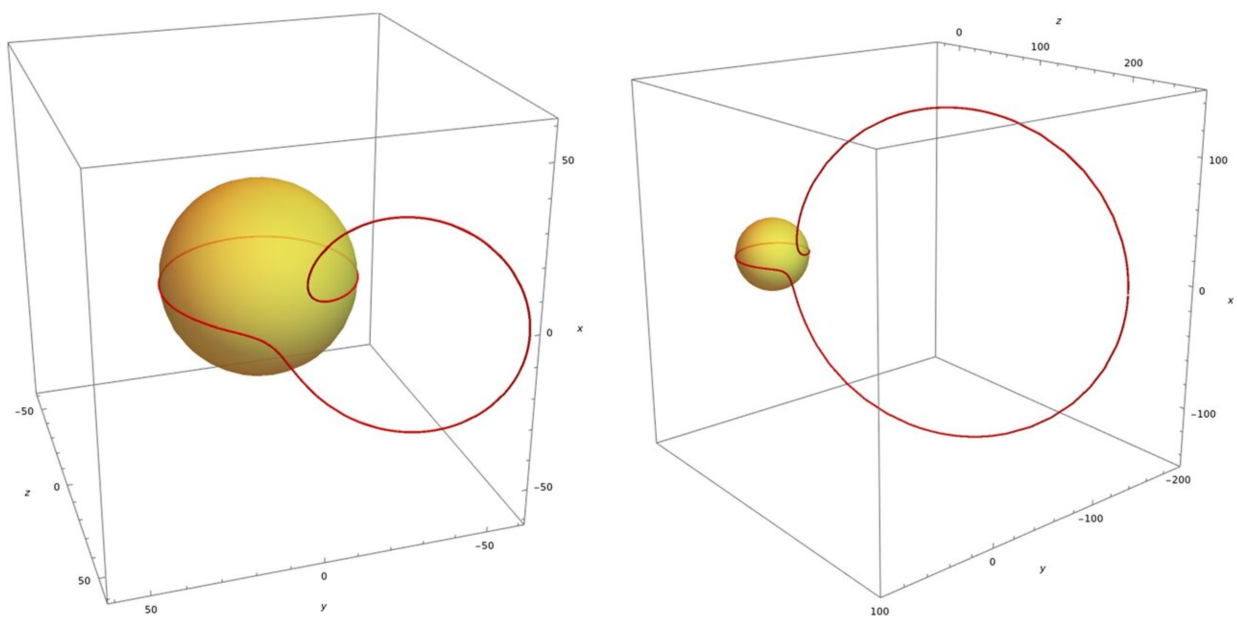
"In our paper, we make a conjecture, that knotted field lines move in a very special manner in which the knotted structure remains," Tselousov continues. "In other words, one can say that this time evolution never involves self-crossings or crossings of two field lines."



The little yellow knot becomes the big red one under time evolution. There are pictures of two successive moments of time. The green curves are the trajectories. The topological structure of knot does not change under the time evolution DOI 10.1140/epjc/s10052-020-08745-7

Tselousov believes that should this conjecture—arrived at with the use of complex computer simulations—be correct, the conservation of the knots implies that their evolution is integrable—capable of undergoing the mathematical function integration. This means that its evolution can be related to other models and systems—in particular with non-linear equations—that are known to share this property.

"It is very rare and always a pleasure to observe the integrable properties of systems because it means deeper understanding and possible further progress. We plan to move in this direction and find more connections with integrability," Tselousov concludes. "In my mind, one of the stunning facts is that light, so familiar to everyone, conceal secrets that we used to ignore for centuries."



The yellow sphere is the light cone x2+y2+z2=t2x2+y2+z2=t2. The red lines are electric field lines at the moment t=30t=30. A part of the field line lies on the equator of the sphere. The other part tends to form a circle DOI

10.1140/epjc/s10052-020-08745-7

More information: A. Morozov et al. Are Maxwell knots integrable?, *The European Physical Journal C* (2020). DOI: 10.1140/epjc/s10052-020-08745-7

Provided by SciencePOD