Six years in 120 pages: Researchers shed light on Ricci flows

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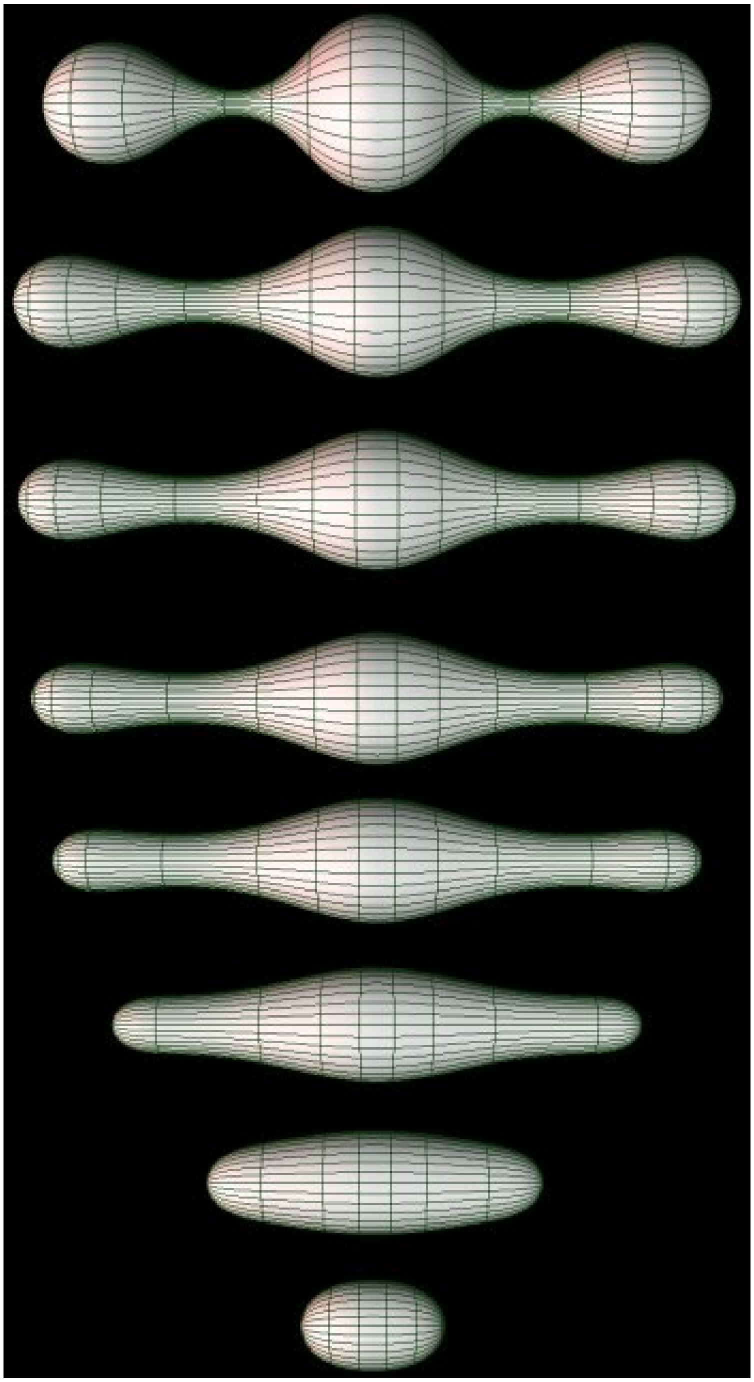

The serval stages of Ricci Flow on a 2D manifold. Credit: https://en.wikipedia.org/wiki/Ricci_flow

Differential geometry is the study of space geometry. Multiple natural phenomena, from universal expansion to thermal expansion and contraction, can come down to spatial evolution. The two core conjectures in this field, the Hamilton-Tian conjecture and the Partial C^0 conjecture, were unsolved puzzles for more than 20 years.

"Most of the pebbles on the beach are round. They might have had edges and corners at first, but as time goes by and the tide ebbs and flows, their shape will get closer and closer to perfection and standard. But no matter how perfect the evolution is, there might still be some abnormalities, which are called 'singularities' in geometry."

"The Hamilton-Tian Conjecture suggests that most of the space is perfect, while the size of the 'singularity' can be restricted to a low-dimensional space," said Prof. Chen Xiuxiong, the founder of the Institute of Geometry and Physics, University of Science and Technology of China (USTC) of the Chinese Academy of Sciences (CAS).

Prof Chen, alongside with Prof. Wang Bing from USTC, first proved the two conjectures.

Their paper was split into 123 pages in two parts of, the first of which was published in 2017 and the second this year on *Journal of Differential Geometry*, which also published Hamilton's fundamental work on Ricci flow after a long course of five years of developing the theory and six years of peer-reviewing since its first submission.

This work emphasized the weak compactness theory for non-collapsed Ricci flows. It introduced many innovative thoughts and methods, which contributed far-reaching implications in the field of geometric analysis, especially for the studies of Ricci flows.

In fact, many other works have been developed based on this article. For example, a new solution for stability of Yau's conjecture based on the structure theory of Ricci flows was given by Prof. Chen, Prof. Wang and Dr. Sun Song of USTC with their derivation published in *Geometry and Topology*. Before that, they received the Oswald Veblen Prize in Geometry for the first solution of the stability of Yau's conjecture.

The theory and methods presented in this article were also applied into a series of works of Prof. Wang and his cooperators in recent years.

The core ideas of this article were generalized to the research of mean curvature flow by Prof. Wang and Prof. Li Haozhao, who solved the extension problem, and the result was published in *Inventions Mathematicae*.

The paper by Prof. Wang, Dr. Huang Shaosai and Dr. Li Yu, "On the Regular-Convexity of Ricci Shrinker Limit Spaces," published in *Crelle's Journal*, has proven that the limit of non-collapsed shrinking Ricci solitons must be the cone shape defined by Prof. Chen and Prof. Wang.

Additionally, the paper "Heat Kernel on Ricci Shrinkers," published in *Calculus of Variations and Partial Differential Equations* by Prof. Wang and Dr. Li, developed several estimates through the study of the heat kernel on Ricci shrinkers and provided "necessary tools to analyze short time singularities of the Ricci flows of general dimension."

This breakthrough was honored by the reviewer of the journal and the winner of Fields Metal, Prof. Simon Donaldson, who said, "this work is a major breakthrough in geometric analysis, and it no doubt will lead many other related research projects."

More information: Xiuxiong Chen et al. Space of Ricci flows (II)—Part B: Weak compactness of the flows, *Journal of Differential Geometry* (2020). DOI: 10.4310/jdg/1599271253

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Xiuxiong Chen et al. Space of Ricci Flows I, *Communications on Pure and Applied Mathematics* (2012). DOI: 10.1002/cpa.21414

Haozhao Li et al. The extension problem of the mean curvature flow (I), *Inventiones mathematicae* (2019). DOI: 10.1007/s00222-019-00893-2

Kähler-Ricci flow, Kähler-Einstein metric, and K-stability. msp.org/gt/2018/22-6/p01.xhtml

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