A stochastic equation for modeling population growth in cities

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A team of researchers at Université Paris-Saclay, CNRS, CEA, has developed a stochastic equation for modeling population growth in cities. In their paper published in the journal *Nature*, the group describes creating their equation to account for "intercity migration shocks" on population changes and the factors that are involved in producing results.

Over the past several hundred years, mathematicians have attempted to create formulas to describe population growth or reduction in major cities in a given country. But so far, the best they have been able to come up with is Zipf's law, also known as the Gabaix model, which takes advantage of the regularity of city growth to estimate future growth. Subsequent efforts using the model have uncovered several flaws, however, particularly when random events occur that can have a dramatic impact on the population of a given city, such as a war.

In this new effort, the researchers have taken some of the important parts of Zipf's law and have added three important factors to introduce randomness: demographics, departures and arrivals, and long-distance migration. They define long-distance migration as movement from rural areas to cities or from one city to another. To create and test their equation, they used city population data from France, the U.K., the U.S. and Canada.

In this effort, they discovered something new about city population growth or decline—migration shocks are important. They define such shocks as rare movements of people into or out of a city due to social, economic or climatic events. They note that history is full of examples of such shocks leading to explosive growth of a city or its demise. Early cities in the American West, they note, are good examples of both. The gold rush in the late 1800s led to fast population growth in some cities and then sudden crashes when the gold ran out.

The researchers suggest their equation can be used by urban planners to estimate city population and distribution—and also to predict variations in a city hierarchy.

More information: Vincent Verbavatz et al. The growth equation of cities, *Nature* (2020). DOI: 10.1038/s41586-020-2900-x