Predicting delayed instabilities in viscoelastic solids

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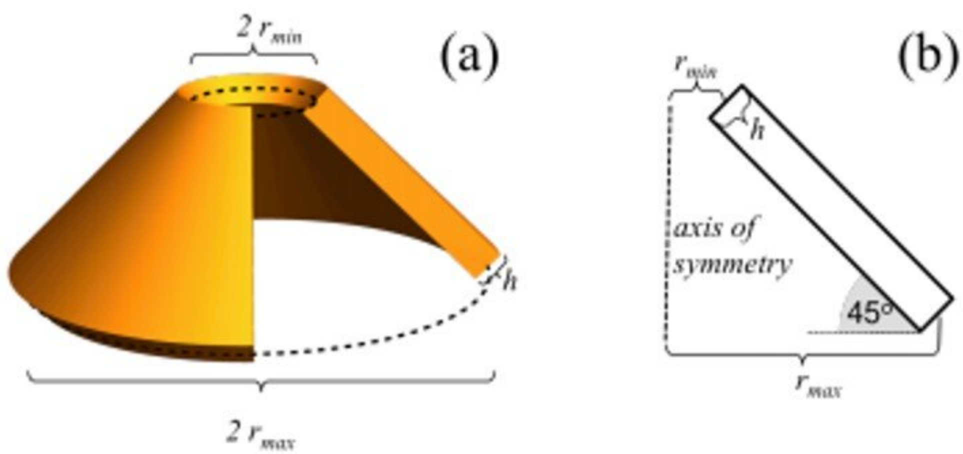
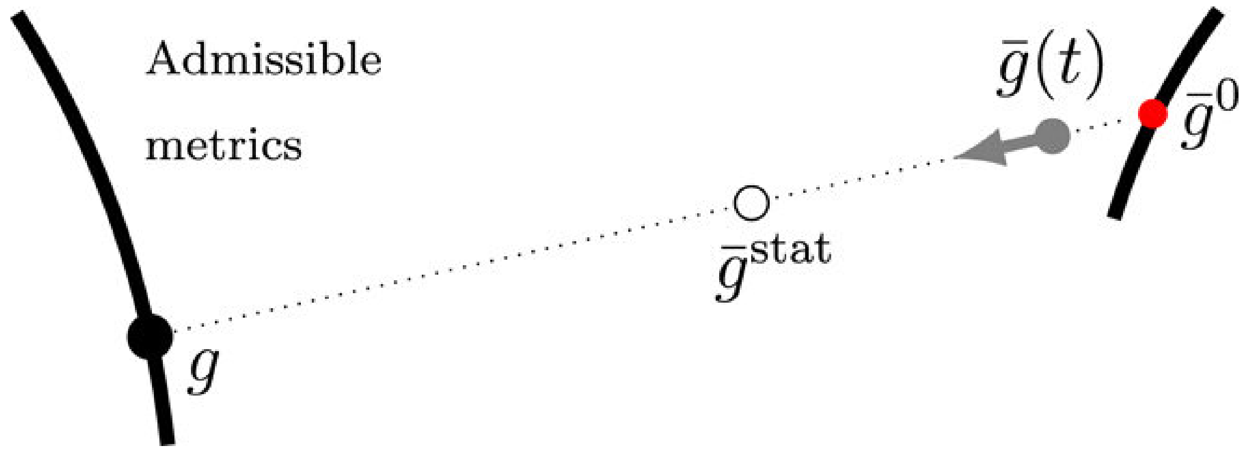


A sketch of the viscoelastic cone, and its dimensions. (a) An angled view with a section of the conical shell cut out. (b) A cross sectional diagram including the axis of symmetry and a single section of the conical shell. Credit: Science Advances, doi: 10.1126/sciadv.abb2948

It is presently challenging to determine the stability of viscoelastic structures since seemingly stable conformations may gradually creep (plastic deformation of a material under stress as a function of time) until their stability is lost. Although a discernable creeping effect does not necessarily lend to instability of viscoelastic solids, researchers are currently limited with numerical simulations to predict the future stability relative to theoretical predictive tools. In a new report on *Science Advances*, Erez Y. Urbach and Efi Efrati in physics and complex systems at the Weizmann Institute of Science, Israel, described viscoelastic solids through an evolving instantaneous reference metric to measure elastic strains. The transparent and intuitive methods derived in this work for incompressible viscoelastic solids reduced the question of future stability to static calculations alone. The team showed the predictive power of the approach by understanding the subtle mechanisms of delayed instability in thin elastomeric shells in order to demonstrate quantitative agreement with experiments.

Creeping motion in nature

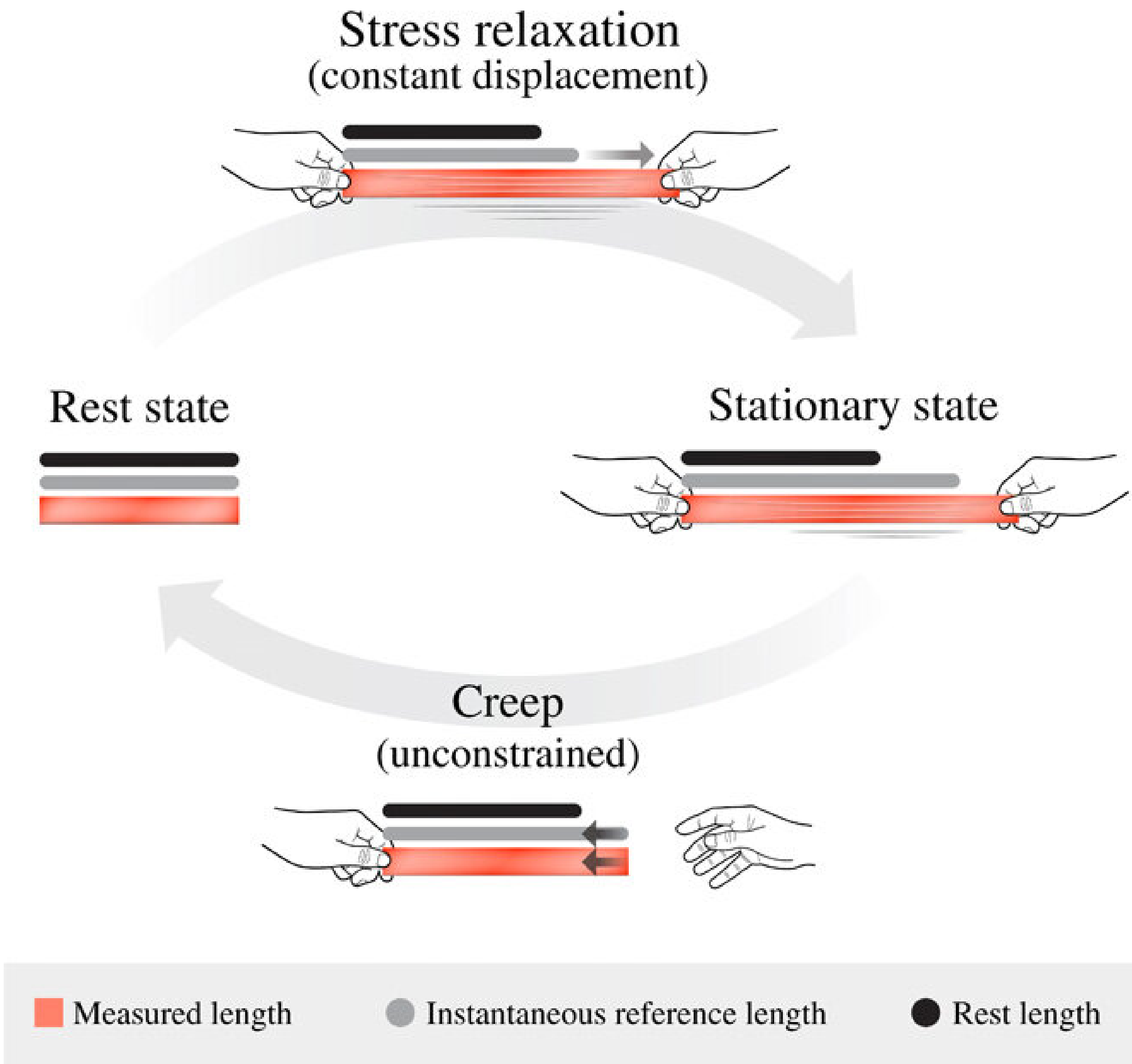
A relatively slow creeping motion underlies the snapping Venus fly-trap – one of the fastest motions in the plant kingdom. Similar creep is observed prior to thin elastomeric shells snapping, known as jumping poppers that last a fraction of a second. While the slow creeping motion of shells appear to be elastically stable, lasting orders of magnitude longer, those on a much larger scale can be noted on Earth's crust before an earthquake aftershock. Researchers are still learning the exact role of viscoelasticity in aftershocks due to the absence of a predictive theoretical framework to detect the future stability of such systems. In each of the outlined examples, the slow viscoelastic flow in the material can lead the system to instabilities, causing an abrupt release of internally stored elastic energy. Although scientists can determine the variables governing viscoelastic behavior, the mechanisms of delayed instabilities in viscoelastic fluids remain poorly understood. In this work, Urbach and Efrati quantitatively addressed the feature of viscoelastic instability by using a metric description.



Schematic representation of the metrics collinearity. The minimization of the metric g (marked by a full black circle) is constrained and performed with respect to the subset of metrics that correspond to realizable configurations (thick black line). These metrics are in, particular, orientation preserving and Euclidean. Given an instantaneous reference metric, g¯ (marked by a full gray circle), the realized metric will correspond to the closest point from the set of admissible metrics to g¯ according to the distance function given by the instantaneous elastic energy. Starting from rest, g¯ evolves from g¯0 (marked by a full red circle) toward the g, which remains the closest admissible metric to g¯ due to collinearity of the three metrics. As g remains stationary, the evolution of g¯ will preserve the collinearity, asymptotically approaching g¯stat (marked by an open circle), which is also collinear. We stress that throughout this evolution, g remains unchanged; thus, no variation of the configuration will be observed despite the stress relaxation. Credit: Science Advances, doi: 10.1126/sciadv.abb2948

Characterizing creeping motion in elastomeric materials

The team described the materials' behavior as a fast elastic response relative to temporally evolving test lengths that can change due to slow viscoelastic flow. They interpreted the microscopic response in the material and predicted the future stability of unconstrained viscoelastic structures. Urbach et al. explained all relations of linear viscoelastic materials through intricate calculations of strain rate with a stress relaxation function, then derived mathematical relationships for one-dimensional systems in this work; some of which depended on material properties such as the Young's modulus and the Poisson's ratio. Instantaneous incremental deformations caused increased linear stress for a purely elastic response in the material. Since viscoelastic materials tend to be dissipative (thermodynamically open), the definition of an elastic free energy can be incomplete. The scientists therefore eliminated inertia from the system and approximated the motion of the material to a quasi-state evolving between elastic equilibrium states. As a result, a given instantaneous reference metric could yield multiple elastically stable configurations.



The viscoelastic reference length evolution. At the resting state, all three length measures on the body, its measured length g (marked red), its instantaneous reference length $\bar{g}$ (marked gray), and its rest reference length $\bar{g}0$ (marked black) are all equal. When subjected to a constant displacement extension, the instantaneous reference length evolves away from the rest length and toward the presently assumed length, thus resulting in stress relaxation. It asymptotically approaches the stationary state $\bar{g}stat=\beta g+(1-\beta)\bar{g}0$, in which the initial stress is reduced by a factor of $1 - \beta$. When released, the unconstrained system immediately adopts its favored instantaneous reference length, which, in turn, gradually creeps toward the rest lengths. Credit: Science Advances, doi:

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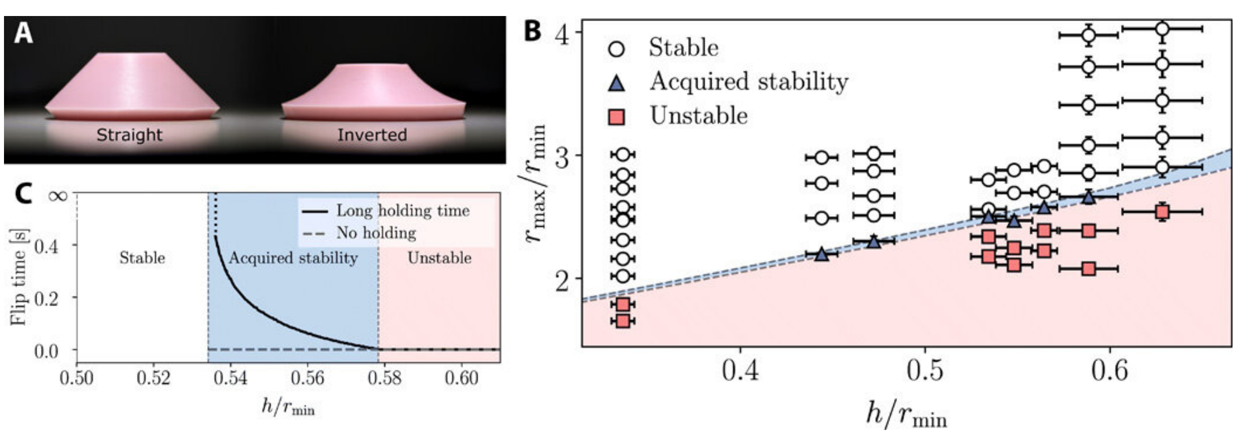
Viscoelastic instabilities through the metric description

The time-dependent instantaneous reference metric of the material could in this way evolve to acquire new stable configurations, merge existing stable points, or cause stable elastic configurations to lose stability. In the latter scenario, the slow viscoelastic evolution will be followed by a rapid snap—highlighting the main difficulty of predicting the stability of viscoelastic structures. This feature is known as temporary bistability, pseudo bistability or creep buckling. Two distinct processes must take place for incompressible linearly viscoelastic solids to creep into instability. First, an elastically stable state will acquire stability through viscoelastic relaxation under some external load for an amount of time. Then as the external load is removed, the body will assume the newly acquired stable state, alongside viscoelastic creep for the resulting instability. However, an acquired stable state is transient (temporary). In this way, Urbach et al. used the metric description of viscoelasticity to provide a picture of the mechanism governing the stability of viscoelastic structures.



Experimental verification of the viscoelastic stability diagram. (A) Straight and inverted conical poppers. Photo credit: Erez Y. Urbach, Weizmann Institute. (B) The two axes span the dimensionless geometrical properties of the truncated conical poppers. The background colors represent the theoretically predicted regions of each of the phases. Each marker corresponds to a different popper; different shaped (and colored) markers indicate the different phases observed in experiment. (C) Numerically calculated flipping time as a function of the normalized thickness of the conical popper for immediate release and long holding time. The different poppers were simulated by varying their thicknesses and constant radii rmin = 10 mm, rmax = 25 mm. The material properties taken were $\beta = 0.1$, and the memory kernel was assumed to be exponential with $\tau = 0.1$ s, Young's modulus E = 2.5 MPa, and Poisson's ratio v = 0.47. Varying the kernel may lead to varying rate of divergence of the flipping time between the stable and acquired stability region, yet the location of this divergence will remain unchanged. The divergence of flipping times was addressed in a previous study, and more recently, the rate of divergence was studied previously as well. Credit: Science Advances, doi: 10.1126/sciadv.abb2948

Experimental results

The calculations conducted in this work revealed many qualitative characteristics of viscoelastic instabilities. The scientists then tested the quantitative predictions of the theory by experimentally examining the response of silicone rubber conical poppers. For this, they cast silicon rubber poppers as truncated conical shells to gain simpler control on the thickness of the material. As the thickness increased, the bistability decreased, then at a timepoint the popper immediately snapped back. The scientists produced 50 different conical poppers of different geometries and tested their phases to experimentally determine the phase boundaries of viscoelastic properties.

The work presented here was similar to previous studies on elastoplasticity. The metric theory can be implemented to isotropic incompressible viscoelastic solids to provide basic rules for viscoelastic instabilities. In order for a given structure to creep into instability, the creeping should have preceded within a timeframe in which the structures were held under an external load. The theory was specifically powerful on application to describe the experimentally delayed instability in thin elastomeric shells. These results will be able to shed light on the role of viscoelasticity in triggering delayed earthquake aftershocks. In this way, the metric description proposed here will provide a theoretical framework to understand delayed viscoelastic instabilities.

More information: Erez Y. Urbach et al. Predicting delayed instabilities in viscoelastic solids, *Science Advances* (2020). DOI: 10.1126/sciadv.abb2948

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