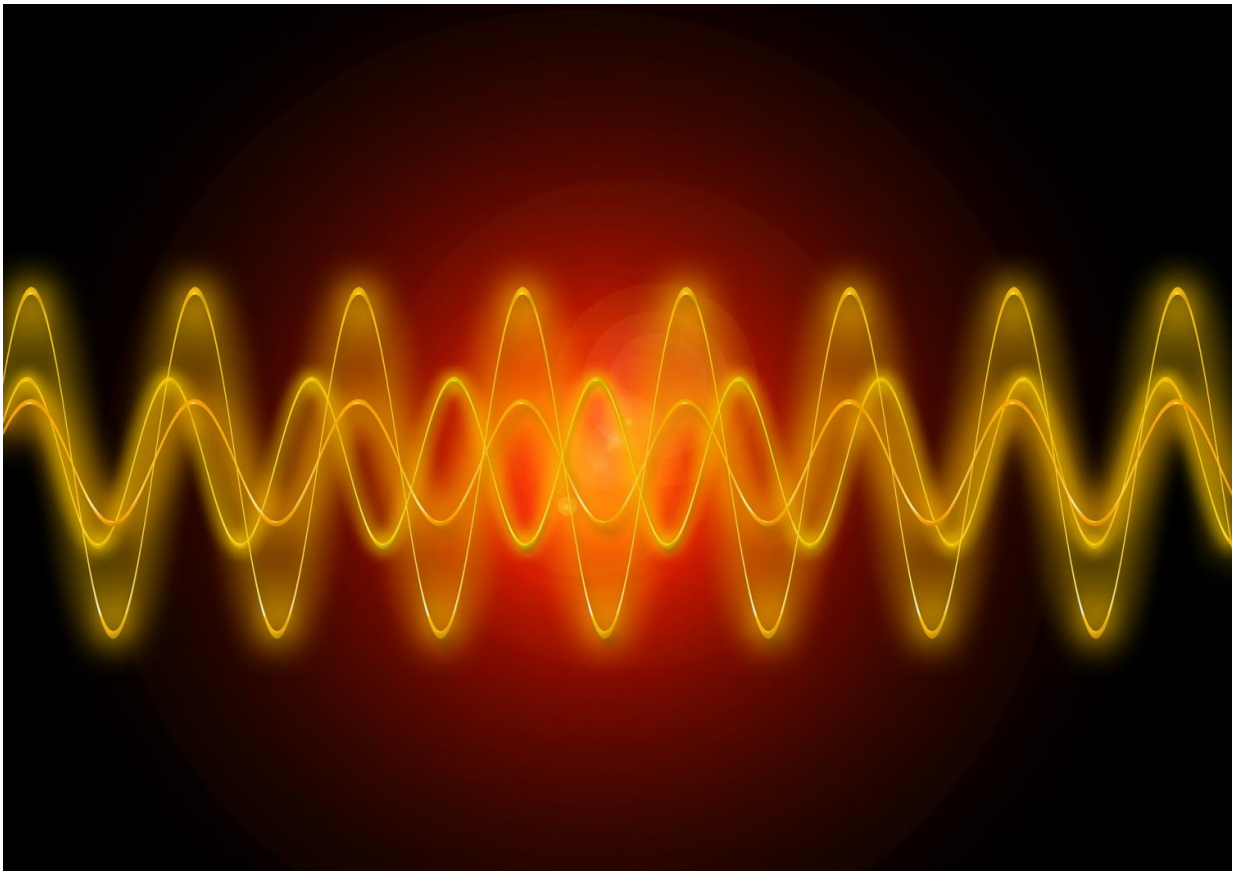
How we 'hear' the shape of a drum

July 14 2020



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How is it that we can recognise the shape of an object despite only seeing a limited range of wave lengths? Radboud mathematician Walter van Suijlekom explains in a new publication in the journal Communications in Mathematical Physics on 14 July. "We can compare

this to MP3s: with a limited range of sound waves, we easily get the impression we are listening to an analogue recording of music." This finding eventually means that we should be able to calculate the shape of our universe.

In 1966, mathematician Mark Kac asked the iconic question: "Can one hear the shape of a drum?" After all, drums of different shapes generate waves at different frequencies and therefore a different sound. Is this information sufficient to be able to determine the shape? At the time, Kac believed it was not, but for the past two years, Walter van Suijlekom and Alain Connes delved deeper into the problem and reached new insights.

Van Suijlekom and Connes, who received the prestigious Fields Medal in 1982, studied how a limited number of vibrations still can provide information about the shape of an object. To this end, they developed a new mathematical theory. This theory describes how, by "listening locally to the shape of the drum," we can estimate its shape very closely.

From analogue to digital

"Let's compare this to MP3s," Van Suijlekom says. "A musical composition is produced by instruments as an analogue signal, with an infinite variation of sound frequencies. However, this analogue signal is digitised by limiting it to a finite number of frequencies; an MP3 file. Still, if we include enough frequencies in the MP3, the difference with the original will dissipate, even to the point of being nearly inaudible."

Estimating the shape of our universe

In the same way that digital MP3s still provide an idea of the analogue source, the limited observations we have of our universe provide

sufficient information to get a better idea of that as well. Using the light frequencies that stars and other celestial bodies emanate, scientists can determine what those objects look like.

Van Suijlekom: "We are developing the mathematics to make this possible. In that, we describe how to calculate the shape of something when only a finite number of its vibrations are available, just like in a physics experiment."

This means that, in principle, it should also be possible to estimate the shape of our universe using a finite number of frequencies. For example, this could be simulated on a computer, something that members of Van Suijlekom's research group are currently working in Nijmegen.

More information: Spectral truncations in noncommutative geometry and operator systems. arxiv.org/abs/2004.14115

Provided by Radboud University