Computational origami: A universal method to wrap 3-D curved surfaces with nonstretchable materials

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Reverse engineering computational origami for conformal wrapping. (A) Wrinkles are formed when tightly wrapping a rectangular sheet of paper around a nonzero Gaussian surface. (B) Fractal cut patterns can avoid wrinkles but
inevitably lead to openings and uncovered areas. (C) The 2D unfolding of a spherical polyhedron generated automatically by computational origami can wrap a steel ball without leaving uncovered areas. (D) As the number of facets increases, the smoothness and conformability of the mesh naturally improve. The difference in surface area between the perfect sphere and the approximated polyhedra decreases by 5.3% when the number of facets increases from 80 to 500. The Hausdorff distance between the polyhedral surfaces and the perfect sphere also reduces from 7.05 to 1.17% of the radius of the perfect sphere when the number of facets increases from 80 to 500 (Photo credit: Y.-K. Lee, Seoul National University). Credit: Science Advances, doi: 10.1126/sciadv.aax6212

The counterintuitive question on how to wrap a curved spherical surface using conventionally stiff and non-stretchable or brittle materials, forms the basis of this study. To answer the question, Yu-Ki Lee and a research team in the departments of materials engineering and computer science in the Republic of Korea and the U.S. extended a geometrical design method of computational origami to wrap spherical constructs in a new report now published in Science Advances. The approach provided a robust and reliable method to engineer conformal devices for arbitrary curved surfaces using a computationally designed nonpolyhedral developable net. The computer-aided design transformed two-dimensional (2-D) materials such as silicon (Si) wafers and steel sheets into conformal structures that could fully wrap 3-D structures without fracture or deformation. The computational wrapping method allowed them to develop a design platform to transform conventionally non-stretchable 2-D devices into conformal 3-D curved surfaces.

The study introduced a universal method for conventional nonstretchable materials to wrap arbitrary and diverse 3-D curved surfaces by engineering conformal material devices without sacrificing their performance. For example, wrapping a sphere with a rectangular piece of paper can inevitably form wrinkles, while attempting to wrap a sphere
with a tougher substrate can cause the wrapping material to fracture. To facilitate the process, materials scientists can introduce patterned cuts into the nonstretchable materials, including lattice cut patterns and fractal cut patterns to effectively wrap 3-D surfaces. Such concepts are shape programmable and can efficiently cover a sphere. Engineers have also recommended computer algorithms to design complex 3-D models based on 2-D auxetic structures. To achieve optimal coverage, they introduced a computational design strategy known as "computational wrapping with nonpolyhedral developable nets," to form nonstretchable material platforms for wearables and conformal devices.

Mathematical limitation of wrapping a planar sheet around a 3D surface with nonzero Gaussian curvatures. (A) Gaussian curvature is the vector product of the maximum and minimum principal curvatures at a point. At the saddle point (black dot) of the gray surface, one of the principal curvatures is the intersection between the red and gray surfaces, and the other is the intersection between the blue and gray surfaces. Both the red and blue planes contain the normal vector of the saddle point, and their intersections with the gray surface define the principal curvatures. A 2D material with zero Gaussian curvature points, such as a sheet of paper, is called a “developable surface”, which cannot be transformed into a 3D surface with a positive or negative Gaussian curvature (i.e., a “nondevelopable
surface”) without stretching or compressing. (B) For example, a cylinder or a cone can be covered with cut paper, but a saddle or a sphere cannot be wrapped without the formation of wrinkles or cuts. The reverse (flattening) process is also the same, which is why there are distortions in the planar map of the Earth.

Credit: Science Advances, doi: 10.1126/sciadv.aax6212

In theory, researchers can characterize a curved surface by the Gaussian curvature – which is the vector product of the maximum and minimum principal curvatures at a point. For example, a sheet of paper is called a 'developable surface' and represents a 2-D material with zero Gaussian curvature at all points. A developable surface cannot be transformed into a nondevelopable 3-D surface without tearing, stretching or compressing the material. The concept is mathematically proven by the "Gauss Theorema Egregium," which states that "To move a surface onto another surface the Gaussian curvature of all corresponding points must match."

Computer scientists have exerted great efforts to algorithmically determine surface cuts that segment a nondevelopable surface into developable surface patches known as polyhedral nets or simply—nets. Recent computational methods aim to optimize net quality and foldability using machine learning methods in order to reduce the time and effort required for traditional trial and error approaches.

Since most real-world 3-D objects are smooth and curved, scientists require high-resolution meshes to cover the surfaces accurately. In this work, Lee et al. developed a new approach known as "computational wrapping" that goes beyond the conventional computational folding method. To accomplish this, they considered conformal device design as a paper wrapping problem instead of a paper folding (origami) challenge. The team recognized the functions of attaching and wrapping conformal devices to cover an underlying curved 3-D surface, simply by bending and pressing a polyhedral net without creases.
Concept and physical demonstration of computational wrapping. (A) When the sum of the folding angles of a net is minimized, the crease lines can be ignored to accommodate flexible but nonstretchable stiff and brittle materials. For 500 meshes, the gaps in the case of a rigid material and the wrinkles in the case of a flexible material are no longer visible, and the difference between the two becomes imperceptible. (B) A nonstretchable stainless steel sheet is cut into a developable net. (C) With a sufficient number of meshes, the stainless steel sheet can be bent and fully wrap a sphere without creasing or folding. (D) Part of the sphere is unfolded with 400 meshes, and the crease lines are removed. (E) A 20-μm-thick brittle Si wafer is cut into an unfolded net with a laser cutter. (F) The cut Si wafer stably wraps both convex and concave frames. (Photo credit: Y.-K. Lee, Seoul National University). Credit: Science Advances, doi: 10.1126/sciadv.aax6212

High resolution meshes allowed them to address the limits of long fabrication times and mechanical reliability. To enclose a surface with all-round non-zero Gaussian curvature, such as a perfect sphere, Lee et al. used a developable surface after refining the facet mesh to meet the required values of wrapping tightness. The results provided data on a nonpolyhedral developable net to create controlled and bound spaces between the net and sphere without gaps or overlaps between the facets. The fabrication process accurately produced highly complex and smooth 3-D surfaces many times faster than conventional computational folding methods when handling complex shapes using paper, metallic and ceramic wrapping materials. Finite element analysis supported that such computational wrappings were mechanically reliable.
Finite element (FE) simulation for wrapping a sphere with a 100 µm-thick Si wafer with a nonpolyhedral developable net. Credit: Science Advances, doi: 10.1126/sciadv.aax6212

The structures developed in the work led to a significant increase in computational origami for real-world industrial fabrication processes. For example, Lee et al. developed a conformal device using electroluminescent lamp (EL) panels to wrap a sphere, the resulting 3-D conformal device exhibited good function and they credited the results to bending and pressing processes used to wrap the sphere instead of creasing and folding techniques. The team also similarly demonstrated their method on a commercial Korean mask and on an electric toy vehicle with attached EL panels to function without failure. To generate the developable net for components with non-zero Gaussian surfaces such as the headlights of the electric toy vehicle, the scientists used the genetic algorithm (GA) unfolding method.
Demonstration of conformal devices. (A) Cuttable, nonstretchable, commercial EL panels consisting of brittle electrodes are cut with a laser cutter to form developable nets for a sphere. (B) EL panels with a developable net can fully cover a sphere and (C) operate without catastrophic failure. (D and E) The computational wrapping concept is also demonstrated for an ellipsoid model. (F and G) In addition to a sphere and an ellipsoid, a commercial Korean facial mask can also be conformably covered with EL panels and operated without electrical failure. (H) An electric toy vehicle can also be conformably wrapped with EL panels in the same manner, and the attached EL panels also operate well without failure. The GA unfolding method is used for generating the developable net for parts with nonzero Gaussian surfaces, including (I) the headlights, the edge of the front side bumper, and (J) the edge of the rear side bumper of the electric toy vehicle. (Photo credit: Y.-K. Lee, Seoul National University.) Credit: Science Advances, doi: 10.1126/sciadv.aax6212

In this way, Yu-Ki Lee and colleagues introduced the concept of computational wrapping to convert nonstretchable 2-D flexible devices into 3-D conformal devices. Using the method, they enclosed a surface with non-zero Gaussian curvature such as a perfect sphere. The proposed technique could control the distance between the two surfaces to ensure tight wrapping. The work produced a single connected surface known as a nonpolyhedral developable net, designed to conformally wrap a 2-D sheet for any 3-D surface. As a result, the scientists were even able to facilitate stiff and brittle materials such as metal sheets and Si wafers to fully cover and wrap non-zero Gaussian curvature surfaces. The universal computational wrapping method developed in this work will provide new insights into the development of conformal devices with arbitrary shapes using efficient algorithms and robust, reliable fabrication methods.


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