Heisenberg limit gets a meaningful update

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Researchers at Warsaw University, Griffith University and Macquarie University have put their heads together to update the Heisenberg limit, a operational consequence of the uncertainty principle. Credit: Gerd Altmann from Pixabay; free for commercial use

One of the cornerstones of quantum theory is a fundamental limit to the precision with which we can know certain pairs of physical quantities, such as position and momentum. For quantum theoretical treatments, this uncertainty principle is couched in terms of the Heisenberg limit, which allows for physical quantities that do not have a corresponding observable in the formulation of quantum mechanics, such as time and energy, or the phase observed in interferometric measurements. It sets a fundamental limit on measurement accuracy in terms of the resources used. Now, a collaboration of researchers in Poland and Australia have proven that the Heisenberg limit as it is commonly stated is not operationally meaningful, and differs from the correct limit by a factor of π.

"The Heisenberg limit can be regarded as a refined variant of the Heisenberg uncertainty relation adapted for the purposes of quantum estimation theory and quantum metrology," explains Wojciech Górecki, the lead author of the *Physics Review Letters* paper recounting this research, alongside Rafał Demkowicz-Dobrzański, Howard Wiseman and Dominic Berry. Quantum metrology exploits quantum effects such as entanglement for high-resolution, high-sensitivity measurements, and as Górecki points out, the Heisenberg limit commonly crops up in this field when dealing with states comprising multiple potentially entangled probes. "Here, the Heisenberg limit indicates a qualitative sensitivity improvement over measurement schemes that do not make use of entanglement."

The Heisenberg uncertainty principle dates back to Heisenberg's work in Copenhagen in 1927, and although radical when it first surfaced, it is now well entrenched in literature and research based on quantum theory. Equally as entrenched, however, is the assumption that boundaries derived from a strand of quantum information theory—quantum Fisher information—can be taken as the actual limits.

From mathematically interesting to operationally meaningful

To understand how Górecki and colleagues arrived at the corrected

Heisenberg limit, consider a probe measuring a system to determine some relevant physical quantity. The value of the quantity is not known before the measurement is taken, and this is formulated by assigning some sort of probability distribution to its value. The Heisenberg limit that has been used so far was based on a "frequentist" approach, whereby only repeatable random events are understood as having probabilities, a definition that excludes hypotheses and fixed but unknown values. As a result, when applying this approach to fixed but unknown physical quantities, the assumption was made that the measurement need only work properly on an infinitesimally small neighborhood of the exact value of the measured quantity. This assumption turned out to be insufficient

To redefine the limit, Górecki and his colleagues adopted a Bayesian approach, which accepts the notion of probabilities representing the uncertainty in any event or hypothesis and attributes a given probability distribution known as the prior, which describes the physical quantity in question. "The Bayesian approach that we follow in this review was often treated as an interesting but somehow artificial approach, as it required a somehow arbitrary choice of the prior," says Górecki. In their report, however, the researchers were able to demonstrate the general relevance of this approach.

When the value of the parameter is assumed to be fixed—the "non-random parameter estimation"—the path the Bayesian approach generally follows can lead to the previously defined Heisenberg limit. However, Górecki and colleagues refined the model to incorporate the fact that as the parameter's value is not known before it is measured, the measurements must work over a fixed region, giving that region a flat prior. This way, no generality is lost by adopting the Bayesian approach. They were also able to exclude some unphysical prior functions like the Dirac delta function, which might lead to arbitrarily high precision.

Previous work had also arrived at the additional factor of π in the Heisenberg limit, but were limited by the assumed Gaussian prior distribution and did not allow for adaptive approaches that achieve a higher-precision result via measured values feeding into future measurements. Having demonstrated the need for an arbitrary but finite prior, Górecki and colleagues were then able to get around a number of other challenges in the way of their final generally applicable result.

Other work and future impact

The Heisenberg limit relates to noiseless systems, which are rare. As a result, the simplicity of using quantum Fisher information to derive the bounds in the standard "frequentist" approach overrode the lack of justification for recklessly taking this bound as the actual limit—most measurements never got close to the limit, anyway.

"Our work is not a harsh criticism of the frequentist approach—it is still a very powerful mathematical tool that we often use ourselves," Górecki points out. "However, one should be aware of its limitations."

As well as their fundamental impact in quantum theory, these results may also affect some areas of practical metrology. In frequency estimation models for estimating atomic frequency transitions and in magnetometry of nitrogen-vacancy centers in diamond (among other studies), the system is probed for a certain length of time rather than by a certain number of photons. "In these setups, it is not unimaginable that the noise in such systems may be low enough, or may be effectively removed by application of quantum error correction-inspired protocols, that the actual precision scaling with the total interrogation time may at sufficiently long (but not too long) times manifest the true Heisenberg limit," says Górecki. With the current interest in quantum error correction-inspired metrological protocols that allow estimation with Heisenberg limit scaling, the results reported here may prove particularly timely.

More information: Wojciech Górecki et al. π-Corrected Heisenberg Limit, *Physical Review Letters* (2020). DOI: 10.1103/PhysRevLett.124.030501