# Mathematician discovers conditions for stabilization of higher-order differential inequalities

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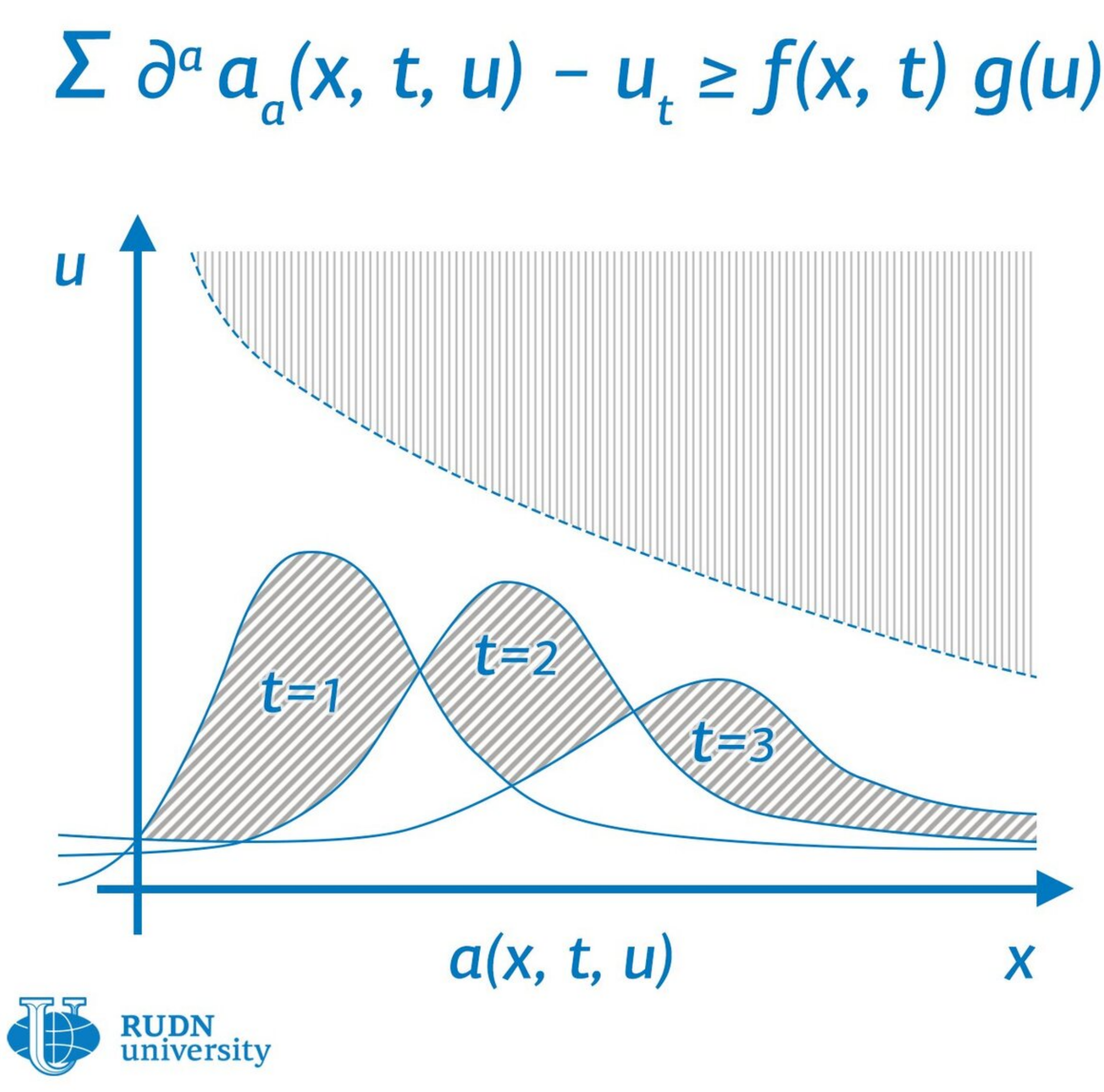

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A RUDN University mathematician (Russia) and a colleague have determined the conditions for stabilization of differential inequalities that have a high order. This result will allow mathematicians to obtain restrictions on the solutions of equations that describe some physical processes, such as diffusion processes and convection processes. The paper is published in the journal *Asymptotic Analysis*.

Interest in differential inequalities arises from a large number of mathematical modeling problems in natural science, as well as in solving technical and physical problems. It is often necessary to define several functions related to several differential inequalities. It is necessary to have the same number of inequalities to do this. If each of these inequalities is differential, that is, has the form of a relation connecting unknown functions and their derivatives, this is a system of differential inequalities. Systems of differential inequalities describe real physical processes with a certain degree of accuracy (for example, devices that record physical phenomena are not perfect and have some errors). It may turn out that a small error in the initial data causes significant changes in the solution of the inequality. Therefore, it is important to set limits on the solutions of differential equations.

Andrey Shishkov from S.M. Nikol'skii Mathematical Institute of RUDN University and Andrej Kon'kov from Moscow State University obtained the result, which generalizes the classical Keller-Osserman condition for differential equations. The Keller-Osserman theorem contains conditions for the absence of positive solutions for second-order nonlinear elliptic inequalities. This theorem serves as the basis for studies of the absence of solutions for equations and inequalities. Moreover, for high-order differential operators, all previously known studies were limited to the case of power nonlinearity. The case of arbitrary nonlinearity has been studied only for second order operators. Mathematicians have researched differential inequalities of higher orders and their result applies to a wide class of problems—equations of the second and third order.

The results can be applied to both parabolic and so-called anti-parabolic inequalities. Parabolic equations are widespread in physics: These include equations that describe the processes of convection, diffusion and its particular case—the heat conduction equation; the Navier-Stokes system of equations that describe the motion of liquids and gases is a system of parabolic equations with divergent constraints.

The questions were previously studied mainly for second-order differential operators, and the case of higher-order operators is much less studied. Mathematicians researched higher-order differential inequalities and obtained sufficient stabilization conditions for so-called weak solutions of differential inequalities. At the same time, the initial conditions are not stipulated on the solutions of the studied differential inequality. The authors also do not stipulate ellipticity conditions on the coefficients of the differential operator.

**More information:** Andrej Kon'kov et al. On stabilization of solutions of higher order evolution inequalities, *Asymptotic Analysis* (2019). DOI: 10.3233/ASY-191522

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