The ever-winning lottery ticket: Mathematicians solve a dusty mystery

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Credit: University of Copenhagen

Is there a lottery ticket that always wins? So goes the popular version of a theoretical conundrum posed in 1969 by English mathematician Adrian R.D. Mathias within the field of set theory, an area dealing with infinity in mathematics.

The problem remained a mystery throughout the 70's, 80's and 90's, as

set theorists the world over tried their best to solve it. Associate Professor Asger Dag Törnquist of the University of Copenhagen's Department of Mathematics was introduced to the problem in 2002 while completing his doctoral dissertation at the University of California, Los Angeles (UCLA).

"Research in the area had gone dormant from the 1990's on because no one was making any progress towards a solution. I was fascinated because it was an old problem that dealt with our understanding of infinity in mathematics. Even then, it became a dream of mine to solve the mystery, even though I had no idea of how to accomplish what had been elusive for others over decades," he says.

MAD families

Mathias researched order and structure, things that occur spontaneously in sufficiently large mathematical systems. Today, this is known as Ramsey Theory, named after British mathematician and philosopher Frank Ramsey. Mathias' research pointed out that there was a profound correlation between Ramsey Theory and what he called MAD families, but he was unable to prove the existence of such a relationship.

"A MAD family can be thought of as a kind of lottery ticket that always wins in a peculiar, infinite lottery game. In this game, lottery tickets have an infinite number of rows of whole numbers, and each row itself has infinitely many numbers. And, a ticket may have so many rows that they simply cannot be numbered," says Törnquist.

What Mathias asked the math world was, 'Does the order and structure that we know is there, as per Ramsey Theory results, prevent the existence of a MAD family, i.e., a ticket that always wins?'

The 'baby-mystery' proved decisive

Törnquist shouldered his dream of solving Mathias' question for several years abroad until he began working at the University of Copenhagen's Department of Mathematical Sciences in 2011. This marked the beginning of a period during which Törnquist and David Schrittesser, his Austrian postdoctoral researcher, would gradually approach the solution.

"In 2014, I decided to rethink the problem from scratch and found a whole new way of tackling it. Alongside the original mystery, Mathias had formulated a sort of baby-version of the mystery. Neither had been solved. I managed to solve the baby version of the mystery, which I then wrote an article about," explains Törnquist.

As a result, a great many mathematicians from around the world reacted. The article suddenly reignited research in the area. Researchers in other parts of the world began to build upon the UCPH researchers' article and more and more pieces of the puzzle began falling into place.

"We were in the midst of writing an article meant to address yet another small piece of the puzzle, when we realized that we may have been closer to solving the entire riddle than we had believed. From then on, things moved quickly. A few weeks later, we had the solution," recounts the mathematician.

Solution: An ever-winning lottery ticket does not exist

After five years of work, Törnquist and Schrittesser had their research article on Adrian Mathias' "lottery ticket" accepted to the prestigious American scientific journal, *Proceedings of the National Academy of Sciences* (*PNAS*). The two researchers discovered that complete coincidence does not exist.

"We found out that lottery ticket numbers clump up in such a way that there is no certainty of a winner, which was what Mathias had guessed would happen, but had been unable to prove. This confirms that one cannot assemble such a type of a lottery ticket without the emergence of certain patterns and regularities in ticket numbers. As such, there is no lottery ticket that always wins Mathias' lottery game," concludes Asger Dag Törnquist.

More information: David Schrittesser et al, The Ramsey property implies no mad families, *Proceedings of the National Academy of Sciences* (2019). DOI: 10.1073/pnas.1906183116

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