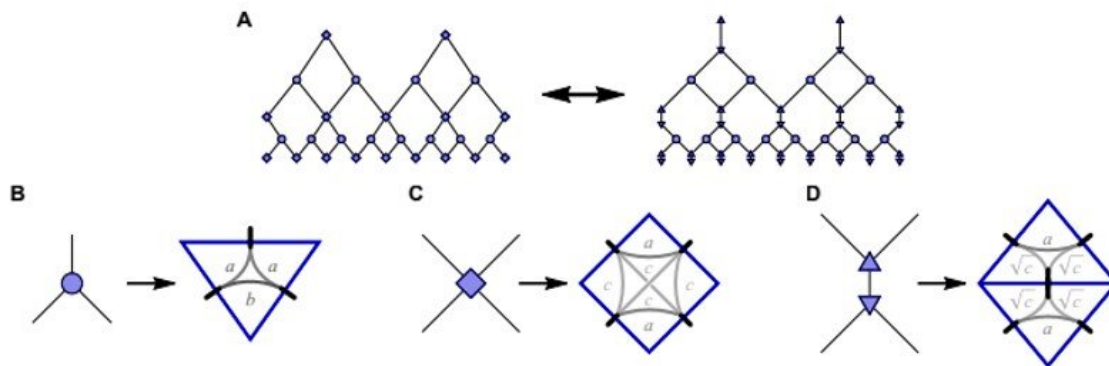


Holography and criticality in matchgate tensor networks

August 19 2019, by Thamarasee Jeewandara



Constructing the matchgate MERA (multiscale entanglement renormalization ansatz simulation; mMERA) toy model. A: The standard MERA tensor network (left) in the numerical matchgate setting of the study is equivalent to B-D: Isometries, disentanglers, and triangulated disentanglers (from left to right) expressed as matchgate tensors. The free parameters a , b , c fix the components of the generating matrices. Credit: Science Advances, doi: 10.1126/sciadv.aaw0092

Tensor networks take a [central role in quantum physics](#) as they can provide an efficient approximation to specific classes of quantum states. The associated graphical language can also easily describe and pictorially reason about quantum circuits, channels, protocols and open systems. In a recent study, A. Jahn and a research team in the departments of

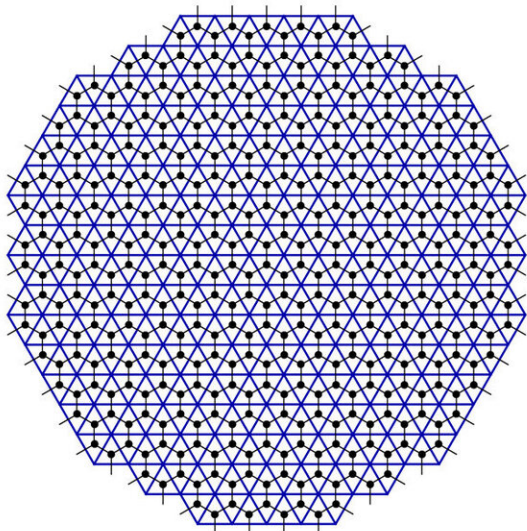
complex quantum systems, materials and energy and mathematics and computer science in Germany introduced a versatile and efficient framework to study tensor networks by extending previous tools. The researchers used [bulk tiling](#) (computing geometric technique) in their work to obtain highly accurate critical data and established a link between holographic quantum error-correcting codes and tensor networks. They expect the work to stimulate further investigations of tensor network models to capture bulk-boundary correspondences. The results are now published on *Science Advances*.

The AdS/CFT correspondence, which stands for anti-de Sitter/conformal field theory correspondence, is one of the largest areas of research in [string theory](#), and is an example in the context of bulk-boundary dualities in which a holographic duality exists between gravity in a bulk space and a critical quantum field on its boundary. This correspondence that relates two very different theories was originally formulated by physicist [Juan M. Maldacena in 1997](#), and is considered a significantly important result in string theory within the last 20 years.

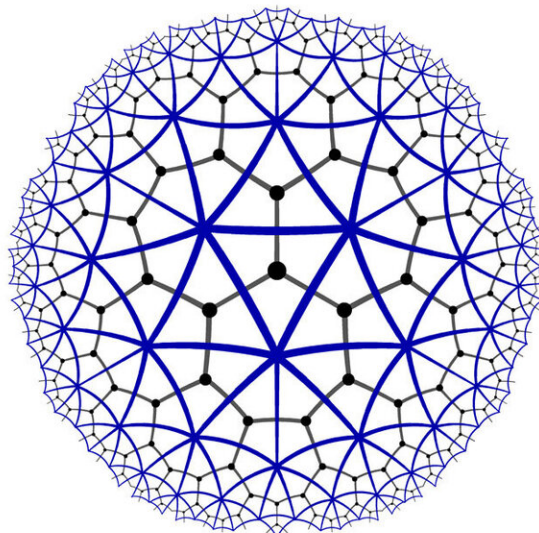
A key feature of these dualities is the relationship between [bulk geometry and boundary entanglement](#) entropies, which physicists had previously illuminated using the [Ryu-Takayanagi](#) formula. Since it is important to understand entanglement in the context of AdS/CFT, researchers realized the necessity of [tensor](#) networks as an ideal framework to construct holographic toy models, such as the [multiscale entanglement renormalization ansatz simulation](#) (MERA). Physicists had previously explored the realization that quantum error correction could be [facilitated by a holographic duality](#), which further connected to ideas from quantum information theory. Although researchers did successfully construct [several tensor network models](#) to reproduce a variety of aspects on AdS/CFT, they still lacked a general understanding of the features and limits of tensor [network](#) holography. Specific obstacles to the process include the potentially large parameter spaces of tensor

networks and the considerable computational costs involved.

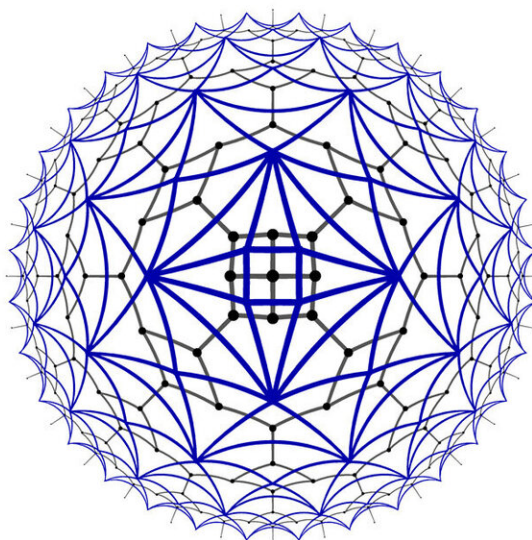
A Flat $\{3,6\}$ tiling



B Hyperbolic $\{3,7\}$ tiling



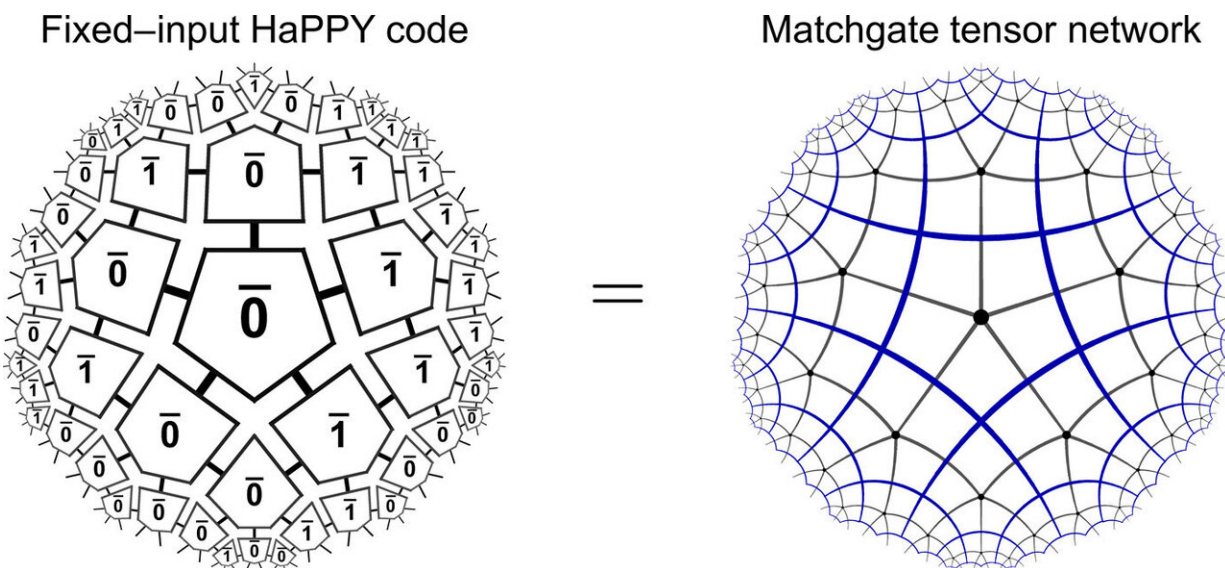
C Hyperbolic mMERA tiling



Geometries of tensor networks. Discretizations of flat (A) and hyperbolic space (B and C) with a triangular tiling (blue edges), into which a tensor network is embedded (black lattice). In the matchgate formalism, joint edges between triangles correspond to an integration over a pair of Grassmann numbers, analogous to tensor network contraction over indices. While (A) and (B) show

regular tilings, (C) presents a nonregular MERA-like tiling, which the scientists named the matchgate MERA (mMERA). Credit: Science Advances, doi: 10.1126/sciadv.aaw0092

In the present work, Jahn et al. overcame the existing challenges by applying highly efficient contraction techniques developed by [matchgate tensors](#). The versatile techniques allowed the research team to comprehensively study the interplay of geometry and correlations in [Gaussian fermionic tensor networks](#) by incorporating toy models of quantum error correction. They also included previous tensor network approaches such as the 'MERA' model within the present work, to highlight connections between them. The team restricted the study to tensor networks that are nonunitary and real, resembling a Euclidean evolution from the bulk to boundary. Jahn et al. provided new approaches in the context of [tensor network renormalization](#), to substantiate the capability of tensor networks to describe bulk-boundary correspondences beyond known models. The present work is preliminary and provides a starting point for more systematic studies on holography in tensor networks.

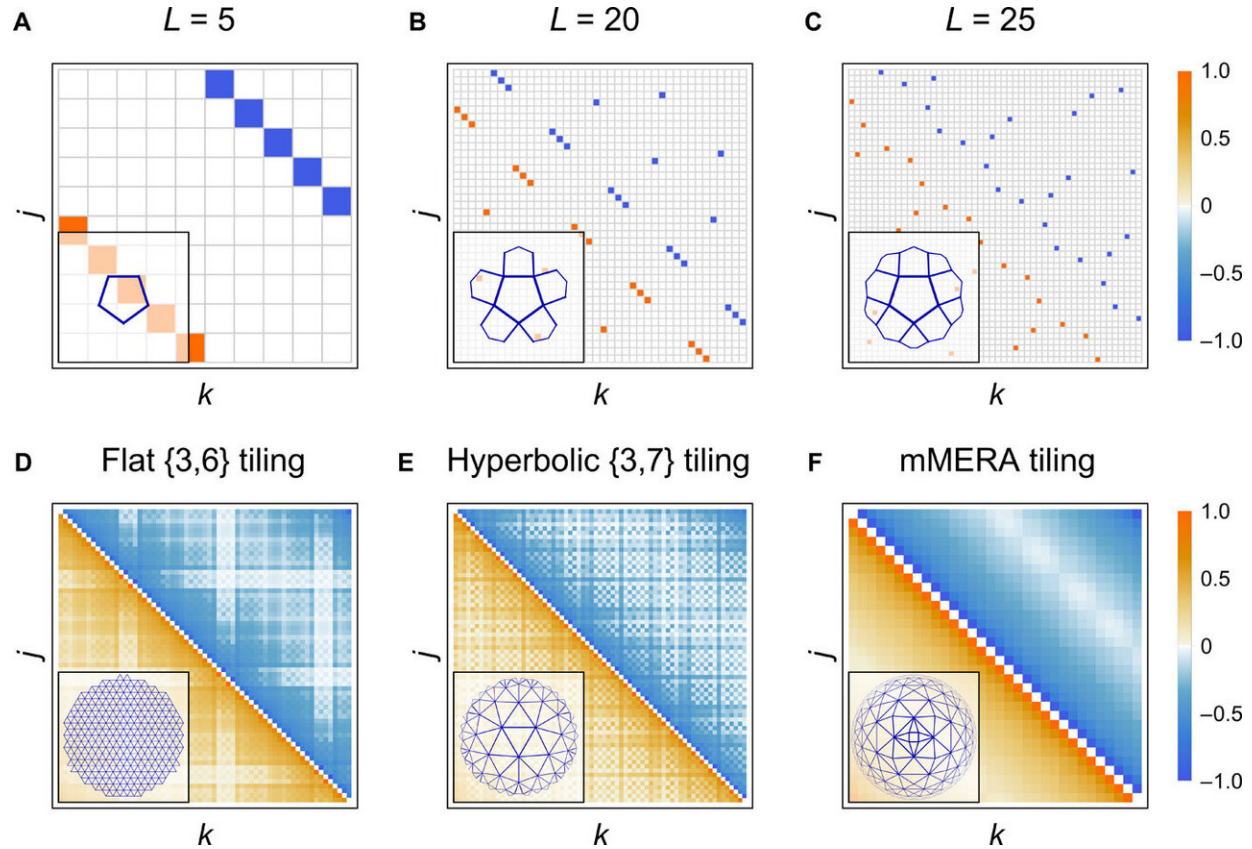


HaPPY/matchgate equivalence. The holographic pentagon code of the HaPPY model for fixed computational bulk input (left) is equal to a matchgate tensor network on a hyperbolic pentagon tiling (right). Credit: Science Advances, doi: 10.1126/sciadv.aaw0092

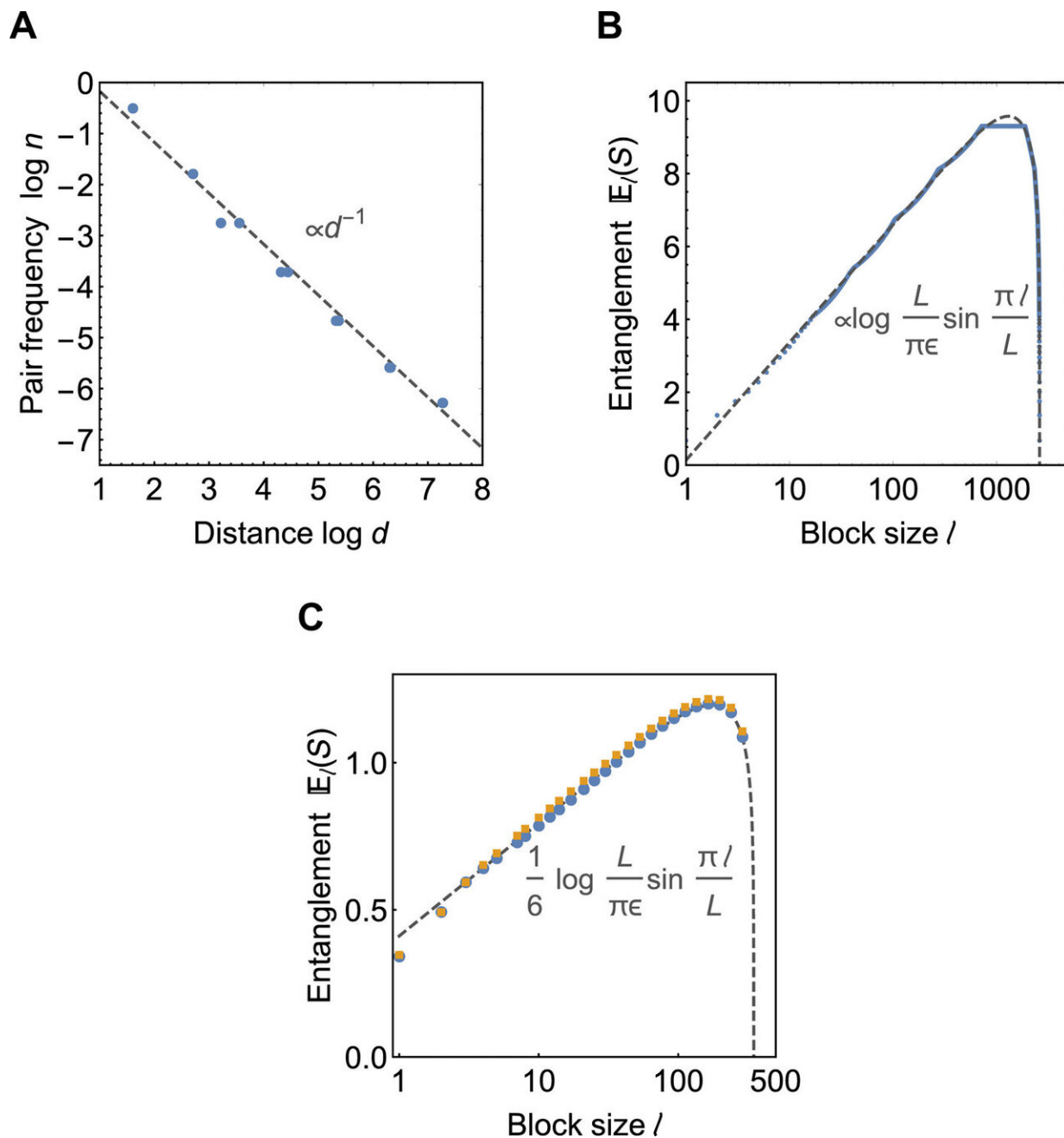
The scientists first applied their framework to the highly symmetric class of regular bulk tilings to implement the [holographic error correcting code \(HaPPY code\)](#) proposed elsewhere. Thereafter, they explored the versatility of the framework to extend it toward more physical setups. They first used the HaPPY code toy model to understand the bulk/boundary correspondence with bulk tiling of holographic pentagons, where each pentagon tile encoded one fault-tolerant logical qubit. Briefly, the research team observed that fixing the bulk degrees of freedom to computational basis states could give rise to a matchgate tensor network. They showed the computational basis states to be pure Gaussian and concluded that for fixed computational input in the bulk, the holographic pentagon code could yield a matchgate tensor on the boundary. Using a [Schläfli symbol](#) $\{p,q\}$ where p = the number of edges per polygon and q = the number of polygons around each corner, they specified the hyperbolic geometry of the HaPPY model.

After Jahn et al. showed their model framework to include the holographic pentagon code built from five-qubit stabilizer states for fixed bulk inputs. They showed the boundary states to correspond to nonlocal bulk pairing with exotic particles known as [Majorana fermions](#). The work thus opened an avenue to study the state properties of a holographic model at large sizes. The scientists further calculated the two-point correlators and [entanglement entropies](#) of the system. They then showed that the critical and gapped Gaussian boundary states could

be realized beyond known models using various bulk tilings. In the present work they reproduced the average scaling properties of the [Ising CFT](#) (conformal field theory) toy model; simplest possible [model](#) in theoretical physics that allowed methods of Euclidean quantum field theory and the study of critical phenomena.



Boundary state correlations. (A to C) Majorana covariance matrix Γ with color-coded entries for a boundary state of a hyperbolic $\{5,4\}$ tiling of the HaPPY code with fixed 0^- input on each tile. Boundary consists of $2L = 10, 40$, and 50 Majorana sites, respectively. (D to F) Field correlation matrix $\langle \psi_j \psi_k \rangle - \psi_j \psi_k / 2 = (\Gamma_{j,2k-1} + \Gamma_{2j-1,2k})/4$ for boundary states of the $\{3,6\}$, $\{3,7\}$, and mMERA tiling at criticality with $L = 63, 69$, and 64 boundary sites, respectively. Matrix entries are normalized to the same color scale. The tiling corresponding to each correlation matrix in (A) to (F) is shown in the lower left corner. Credit: Science Advances, doi: 10.1126/sciadv.aaw0092



Critical correlations and entanglement scaling. (A and B) Boundary state properties of the HaPPY code at 2605 boundary sites. (A) shows average correlations at boundary distance d , computed as the relative frequency n of Majorana pairs. Dashed gray line shows an $n(d) \sim 1/d$ numerical fit. (B) shows the scaling of average entanglement entropy $E_l(S)$ with subsystem size l . Dashed

gray line shows numerical fit using (11). (C) $\text{El}(\text{S})$ for regular tilings at the critical values $a = 0.580$ for a $\{3,6\}$ tiling (blue) and at $a = 0.609$ for the $\{3,7\}$ tiling (yellow) with 348 boundary sites each. The dashed gray line shows the exact $c = 1/2$ CFT solution. Credit: Science Advances, doi: 10.1126/sciadv.aaw0092

Jahn et al. then constructed a Euclidean matchgate tensor network based on the previously developed MERA geometry and named it the matchgate MERA (mMERA). This tiling invariance which they expressed as a [triangulation](#) (multiple measures to capture a construct), recovered the Ising CFT with little computational cost. The computational optimization process in the study only took a few minutes on a desktop computer for a network with hundreds of tensors.

In this way, A. Jahn and colleagues introduced an efficient preliminary framework to study tensor networks and proposed for further studies within the Gaussian setting to focus on positively curved bulks, higher dimensional models and random tensors. Additional studies beyond Gaussianity could explore interacting fermionic tensor networks by weak-coupling expansion or under locally restricted interactions. Both of the suggested possible extensions of the framework presented in the study will only require computational scaling polynomial to the system size to avoid prohibitive computational efforts of general methods to extract tensor contraction.

More information: A. Jahn et al. Holography and criticality in matchgate tensor networks, *Science Advances* (2019). [DOI: 10.1126/sciadv.aaw0092](https://doi.org/10.1126/sciadv.aaw0092)

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