General approach for the solution of lattice gauge theories

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It is not the daily occurrence that physicists from entirely different fields closely work together. However, in theoretical physics a general ansatz can offer solutions for a large variety of problems. A team of scientists from the Theory Division of Professor Ignacio Cirac at the Max Planck Institute of Quantum Optics has now for a couple of years collaborated with theorists from the field of particle physics, in order to find a new and simplified formulation of lattice gauge theories. (*Physical Review X* 7, 28 November 2017)

Gauge theories play a central role in many areas of physics. They are, for instance, the foundation of the theoretical description of the standard model of particle physics that has been developed in the 1970ies. In this theory, both the elementary particles and the forces that act between them are described in terms of fields, whereby gauge invariance has to be ensured: different configurations of these fields, which can be transformed into each other by generalized local rotations—so-called gauge transformations—should have no impact on related observable quantities such as the mass or charge of a particle or the strength of the interacting force. In the theoretical description, this local symmetry is ensured by introducing additional degrees of freedom in form of a gauge field. These degrees of freedom, however, are often partially redundant, rendering gauge theories very difficult to solve.

"It is our goal to find a formulation, i.e. the Hamiltonian of the system, which minimizes the complexity of its description. As a prototype, we take a special gauge system with only one dimension in space and time," explains Dr. Mari Carmen Bañuls, a senior scientist in the Theory Division of Professor Ignacio Cirac. For the simple case of one temporal and one spatial dimension, the gauge degrees of freedom are not truly independent and can in principle be integrated out, so it should be possible to find a description that does not require additional gauge degrees of freedom. At first sight, this makes these systems simpler to work with. "However, this approach has so far only been successful for Abelian gauge theories, the most simple case, in which gauge fields only interact with matter fields and not with themselves," Dr. Bañuls elaborates. "For non-Abelian theories like the ones that arise in the standard model the self-interaction of the gauge fields makes things much more complicated."

A fundamental tool for the numerical study of gauge models is lattice gauge theory. Here, the space-time continuum is approximated by a lattice of discrete points, still ensuring gauge invariance. Based on a lattice formulation the scientists have developed a new formulation of a non-Abelian SU(2) gauge theory in which the gauge degrees of freedom are integrated out. "This formulation is independent of the technique that is used to calculate the energy eigenstates of the systems. It can be used for any numerical or analytical method," Dr. Stefan Kühn emphasizes who has worked on this topic for his doctoral thesis and is at present postdoc scientist at the Perimeter Institute for Theoretical Physics in Waterloo (Ontario, Canada). "However, we found out, that this formulation is especially well suited to solve the lattice gauge model with tensor networks."

The method of tensor networks has originally been developed by the MPQ scientists for the description of quantum many-body-systems in the context of quantum information theory. "Compared to other methods, tensor networks offer the advantage of providing information about the entanglement structure of the system," Mari Carmen Bañuls points out. "The direct access to the quantum correlations in the system offers new possibilities to characterize lattice gauge theories." And Stefan Kühn summarizes the versatility of the new method. "On the one hand, our formulation of a low-dimensional gauge theory makes it easier to calculate and predict certain phenomena in particle physics. On the other hand, it might be suited to design quantum simulators for applications in quantum computing."

More information: Mari Carmen Bañuls et al. Efficient Basis Formulation for (1+1)-Dimensional SU(2) Lattice Gauge Theory: Spectral Calculations with Matrix Product States, *Physical Review X* (2017). DOI: 10.1103/PhysRevX.7.041046

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