Mathematicians obtain new fundamental results in functional inequalities

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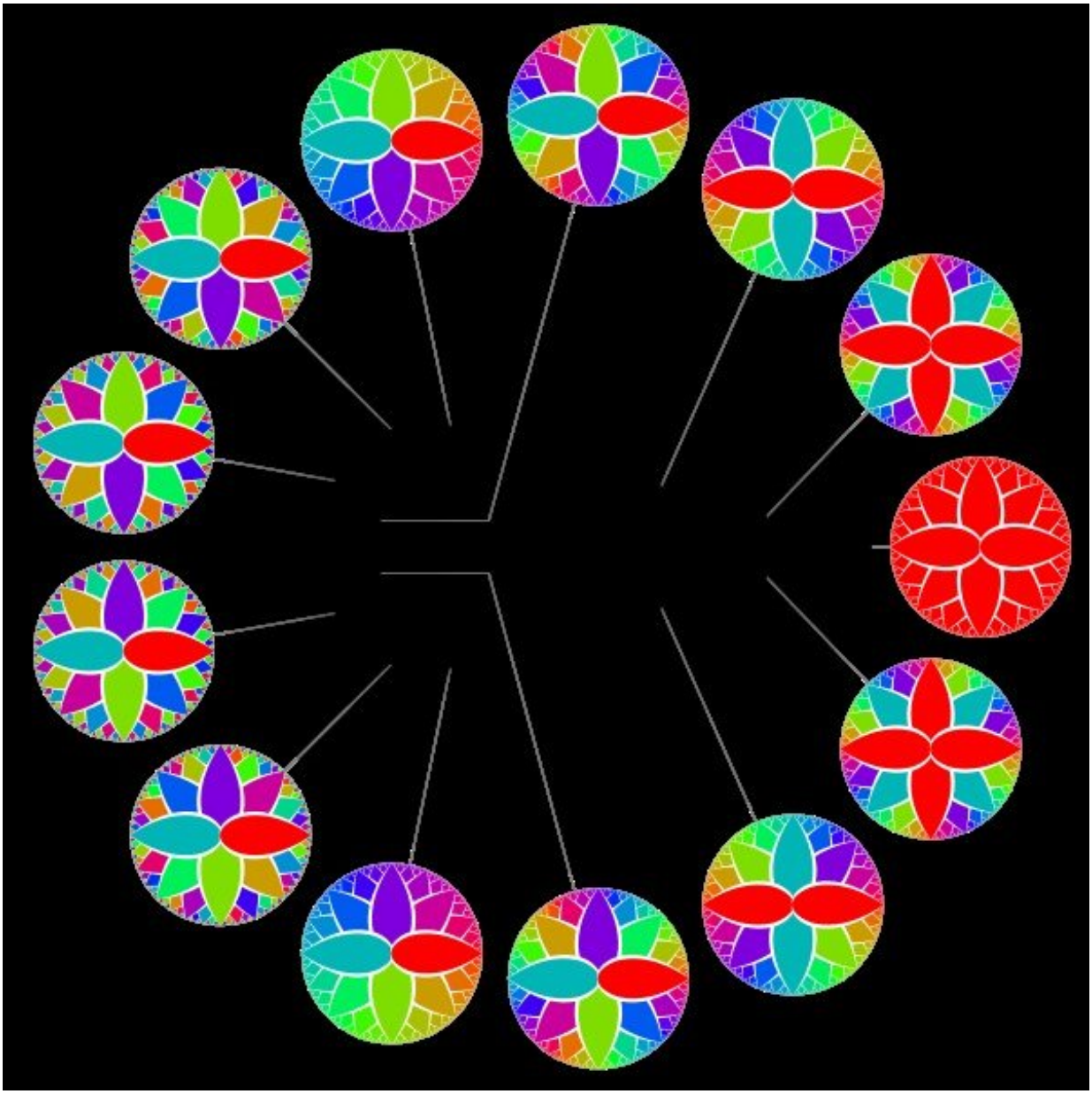

Credit: RUDN University

Invited professor of RUDN University Durvudkhan Suragan and a team of colleagues have obtained and established new types of functional inequalities. Hardy's inequalities are an important type of problem solving in mathematical physics. The results of the study were published in *Advances in Mathematics*.

The properties of the so-called Hardy's inequalities have been studied by mathematicians all over the world for about a century. They are relations of a certain type for series and integrals. Hardy's inequalities are studied in functional analysis and used as an auxiliary instrument in many areas of mathematics and mechanics, as well as in the degenerate differential equations theory (in elliptic-type partial derivatives), spectrum theory, nonlinear analysis and interpolation theory.

The majority of studies covering Hardy's inequalities and their analogs are carried out in Euclidean vector spaces. From the point of view of higher mathematics, a Euclidean space is a set of arbitrary elements on which a dot product operation is given. Two- and three-dimensional spaces are special cases of Euclidean spaces. A team from RUDN extended the theory of Hardy-type inequalities and studied them in terms of more complicated mathematical objects—homogeneous topological groups.

A set of elements is called a topological group if it is a topological space and a group at the same time, and the operations of product and inverse element derivation are continuous. A system of subsets (topology) of special properties is found in a topological space. Besides the subsets themselves, topology includes their aggregates consisting of arbitrary number of element, as well as intersections (only the finite ones), and

void sets. The presence of a group structure means that an associative algebraic operation is given for the set, it contains the so-called "figure of one" (the one having the properties of 1 in multiplying), and all elements have inverse ones.

Existing methods of establishing functional inequalities in homogenous topological groups are based on studying the properties of norms. A norm in mathematics is a non-negative composite function that meets certain requirements. Number module and vector length are simple examples of norms. New methods suggested by the authors of the study allow working with random norms, not strictly determined and fixed composite functions that were used before.

The result of the team's work was obtaining and establishment of new types of Hardy's inequalities in homogenous groups. Special attention was given to analysis in Abelian groups. Abelianness (or commutativity) is expressed in the independence of a group operation result from the order of the elements. A specific case of commutativity is the well-known rule "permuting the summands of a sum does not change the value of the sum." Scientists point out that the newly obtained inequalities may be used in the nonlinear differential equations theory.

The results of the study are mainly theoretical and fundamental. Existing results of Hardy-type inequalities analysis have been reconsidered and expanded to new classes of mathematical objects. Therefore, further unknown applications for these inequalities may be discovered.

More information: Michael Ruzhansky et al. Hardy and Rellich inequalities, identities, and sharp remainders on homogeneous groups, *Advances in Mathematics* (2017). DOI: 10.1016/j.aim.2017.07.020

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