Mathematician's study of 'swarmalators' could direct future science

November 17 2017, by Tom Fleischman

How does the Japanese tree frog figure into the latest work of noted mathematician Steven Strogatz? As it turns out, quite prominently.

"We had read about these funny frogs that hop around and croak," said Strogatz, the Jacob Gould Schurman Professor of Applied Mathematics. "They form patterns in space and time. Usually it's about reproduction. And based on how the other guy or guys are croaking, they don't want to be around another one that's croaking at the same time as they are, because they'll jam each other."

Strogatz and Kevin O'Keeffe, Ph.D. '17, used the curious mating ritual of male Japanese tree frogs as inspiration for their exploration of "swarmalators" - their term for systems in which both synchronization and swarming occur together.

Specifically, they considered oscillators whose phase dynamics and spatial dynamics are coupled. In the instance of the male tree frogs, they attempt to croak in exact anti-phase (one croaks while the other is silent) while moving away from a rival so as to be heard by females.

This opens up "a new class of math problems," said Strogatz, a Stephen H. Weiss Presidential Fellow. "The question is, what do we expect to see when people start building systems like this or observing them in biology?"

Their paper, "Oscillators That Sync and Swarm," was published Nov. 13

in *Nature Communications*. Strogatz and O'Keeffe - now a postdoctoral researcher with the Senseable City Lab at the Massachusetts Institute of Technology - collaborated with Hyunsuk Hong from Chonbuk National University in Jeonju, South Korea.

Swarming and synchronization both involve large, self-organizing groups of individuals interacting according to simple rules, but rarely have they been studied together, O'Keeffe said.

"No one had connected these two areas, in spite of the fact that there were all these parallels," he said. "That was the theoretical idea that sort of seduced us, I suppose. And there were also a couple of concrete examples, which we liked - including the tree frogs."

Studies of swarms focus on how animals move - think of birds flocking or fish schooling - while neglecting the dynamics of their internal states. Studies of synchronization do the opposite: They focus on oscillators' internal dynamics. Strogatz long has been fascinated by fireflies' synchrony and other similar phenomena, giving a TED Talk on the topic in 2004, but not on their motion.

"[Swarming and synchronization] are so similar, and yet they were never connected together, and it seems so obvious," O'Keeffe said. "It's a whole new landscape of possible behaviors that hadn't been explored before."

Using a pair of governing equations that assume swarmalators are free to move about, along with numerical simulations, the group found that a swarmalator system settles into one of five states:

- Static synchrony - featuring circular symmetry, crystal-like distribution, fully synchronized in phase;
- Static asynchrony - featuring uniform distribution, meaning that

every phase occurs everywhere;
- Static phase wave - swarmalators settle near others in a phase similar to their own, and phases are frozen at their initial values;
- Splintered phase wave - nonstationary, disconnected clusters of distinct phases; and
- Active phase wave - similar to bidirectional states found in biological swarms, where populations split into counter-rotating subgroups; also similar to vortex arrays formed by groups of sperm.

Through the study of simple models, the group found that the coupling of "sync" and "swarm" leads to rich patterns in both time and space, and could lead to further study of systems that exhibit this dual behavior.

"This opens up a lot of questions for many parts of science - there are a lot of things to try that people hadn't thought of trying," Strogatz said. "It's science that opens doors for science. It's inaugurating science, rather than culminating science."

More information: Kevin P. O'Keeffe et al. Oscillators that sync and swarm, *Nature Communications* (2017). DOI: 10.1038/s41467-017-01190-3

Provided by Cornell University