Researchers find optimal rules for seedings in knock-out tournaments

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Researchers from the Higher School of Economics and the Stanford Graduate School of Business have conducted a study on tournaments using the playoff system, which is one of the most popular forms of sporting competitions. In the playoff system, two teams play one another in each match, and the winner advances to the next stage of the tournament, while the loser is eliminated. Mathematical calculations allowed the researchers to identify two optimum systems for solving the discrete optimization problem of finding a seeding that maximizes spectator interest in a tournament when spectators are interested in matches with a high level of competitiveness. The results of the study were published in the *Journal of Combinatorial Optimization*.

Many competitions are held in the playoff framework and see the participation of a large number of teams or individual players. Examples of such tournaments include the national football cups held in different countries around the world. The tournament organisers usually want the competition to be as interesting as possible. Empirical studies show that there are two key parameters that influence interest in a specific match - the quality of the match, or the level of each side's abilities, and the competitiveness of the match. The closer the opponents' abilities are, the less predictable a match is and the higher the demand is to view the match.

In order to prevent the stronger teams from meeting one another and being eliminated in the early stages of the competition, a seeding system is used in many playoff tournaments. This simply means that the

strongest participants are kept separate from one another in fixed positions on the tournament bracket. The Wimbledon tennis tournament, for example, seeds 32 players out of 128. The first participant is on the first line of the bracket, the second is on the last line, so that two best players do not meet before the final. The third participant is seeded first in the lower half of the bracket, and the fourth is last on the upper half of the bracket, and so on. This is the traditional seeding model used in many tournaments. HSE researchers asked the question of whether this seeding system really maximises overall spectator interest in the tournament. They tasked themselves with looking at all possible seedings to find the one that maximises overall spectator interest in all matches in a tournament.

'Together with Alex Suzdaltsev of the Stanford Graduate School of Business, we were able to prove that if the demand to view a match is linearly dependent on the quality of a match and on its competitiveness, with stronger participants beating weaker ones with a sufficiently high probability, then there are only two possibilities when it comes to optimal seeding systems. The first is traditionally used in tournaments like Wimbledon. We call this system 'distant seeding,' which is when each team from the upper half plays a team from the bottom half each round. This system is useful for organisers if they particularly value (in terms of ticket prices, for example) the final match versus the matches from the first round,' says Dmitry Dagaev, associate professor in the HSE Department of Higher Mathematics.

'Conversely,' he says, 'if the final is just as important to the organisers as the matches from the initial stages, then they should stick to a different seeding system. More specifically, in each round the strongest team should play the team that is the second strongest, while the third plays the fourth strongest, the fifth - the sixth, etc. We call this a 'close seeding' system. It turns out that there are no optimal seedings aside from close and distant, despite the fact that there are many different

ways to distribute teams in a bracket.'

The researchers' theoretical model was based on two assumptions: the objective function is linear in quality and competitive intensity and a stronger team beats a weaker one with sufficiently high probability. These assumptions were enough to show that only two types of seedings are optimal. The researchers note that they are not aware of tournaments that use a close seeding system, which is why the results of the study may indicate that tournament organisers have underestimated this system or have not considered it whatsoever.

A number of studies have already been conducted on determining optimal seeding systems, but the results of these studies have generally concerned tournaments with a lower number of players. This is related to the rapid growth in the number of possible seedings depending on the number of players in a tournament. This work is unique in that these results are valid for tournaments with an arbitrary large number of participants. In the future, the researchers plan to continue studying and comparing various tournament formats.