Study unravels long-held Fermi puzzle tied to nonlinear systems

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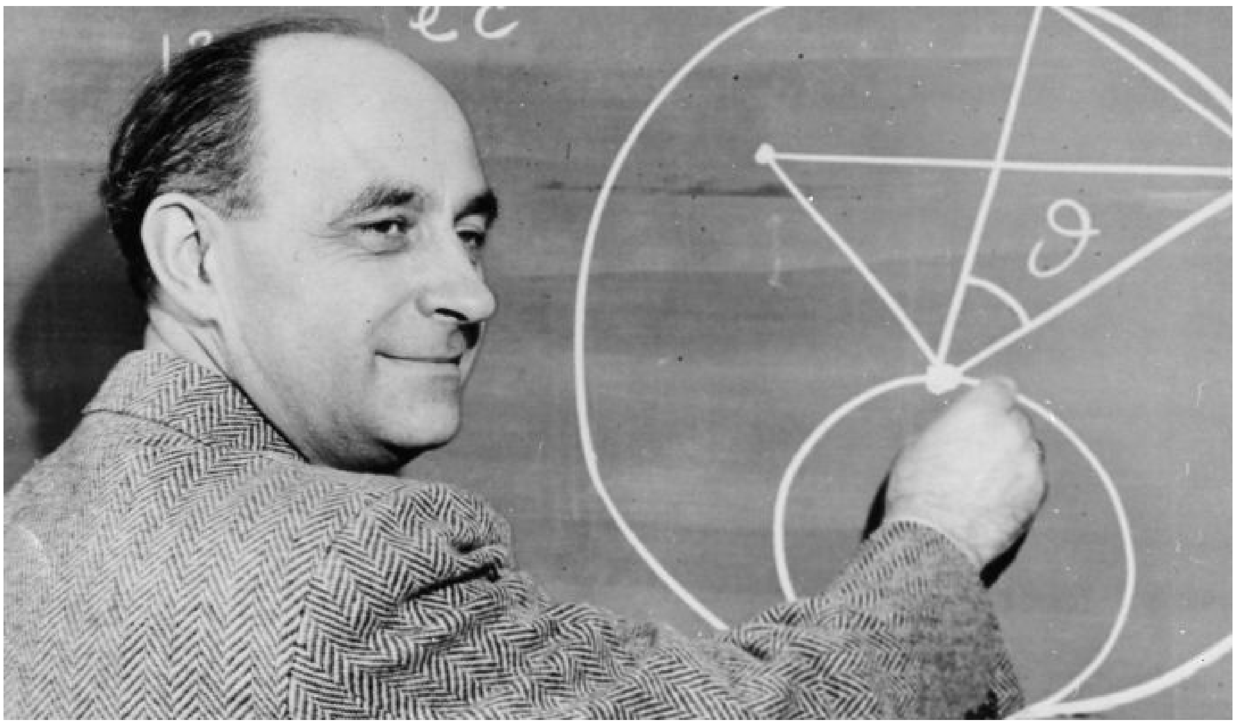

Enrico Fermi at the blackboard. Credit: Wiki Commons.

In physics, the Fermi-Pasta-Ulam-Tsingou (FPUT) problem—which found that certain nonlinear systems do not disperse their energy, but rather return to their initial excited states—has been a challenge that scientists have tackled repeatedly since 1955.

The challenge within the FPUT problem was that the scientists expected the system to achieve a relaxed state, possibly equilibrium, but instead it never relaxed.

Numerous papers have narrowed the focus of the problem, finding that weak nonlinear systems can reach a type of equilibrium. But the question of strongly nonlinear systems reaching full equilibrium has remained a mystery.

Now, a discovery by an international team of scientists, published in March in the journal *Physical Review E*, has found that such a system can reach equilibrium, provided certain conditions are met.

"That is a big deal," said Surajit Sen, PhD, a physics professor in the University at Buffalo's College of Arts and Sciences and co-author of the paper, "because in a very convoluted way, it confirms what [Enrico] Fermi had thought probably should happen."

Sen has been studying solitary waves, generated in a chain of solid spheres—or grains—held between stationary walls, for more than two decades. In 2000, he discovered how such waves can break into smaller "baby" solitary waves. Further research by others found that these solitary waves, under certain conditions, could reach a state of quasi-equilibrium, a generally calm state, but with large kinetic energy fluctuations.

Yet whether these strongly nonlinear systems could relax beyond this quasi-equilibrium phase, where the large kinetic energy fluctuations settle to much smaller equilibrium values, remained uncertain.

"What we are finding is that when these solitary waves continuously break down during collisions, they start to break down and reform. When this breaking down and reforming become comparable, that's when you get to the quasi-equilibrium phase," Sen said.

When the number of solitary waves running around the system become too large to even count, that is when the quasi-equilibrium ever-so-slowly goes over to true equilibrium where energy is roughly equally shared by all the particles.

Sen concedes that it is reasonable to ask: What does it matter? On one level, Sen says, this is pure science, with few immediate practical applications. However, there may be practical applications for materials science.

"I think it has implications in materials modeling," Sen said. "Suppose I want to make a material capable of withstanding enormous amounts of heat, or one that converts a mechanical vibration to electrical current. To make them, I have to have a really good understanding of how these materials transfer energy, and this research cuts right to the heart of it."

The breakthrough in the research came when Michelle Przedborski, a PhD student at Brock University in Canada, examined the specific heat of the chain of solid spheres by considering the collisions between the spheres. The specific heat behavior and the energy fluctuation, due to the collisions as predicted by the equilibrium theory, agreed exactly with the results predicted by dynamical computer simulations.

"That was the 'aha!' moment," Sen said. "They come from two different routes. Nothing can be sweeter than this, because when you have an agreement of this magnitude and of this level of exactness, you know the system is in equilibrium. There are no 'if, ands or buts' about it.

"What we have managed to show—in the context of the Fermi-Pasta-Ulam-Tsingou problem, where the question was raised whether non-linear systems would go to equilibrium, over which there has been this

60-plus year debate—is that strongly non-linear systems such as these do go to equilibrium."

Among the conditions required for the equilibrium state to be reached are that solitary waves must interact, or collide with each other, and the system must be gently perturbed, rather than violently shaken.

More information: Michelle Przedborski et al, Fluctuations in Hertz chains at equilibrium, *Physical Review E* (2017). DOI: 10.1103/PhysRevE.95.032903

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