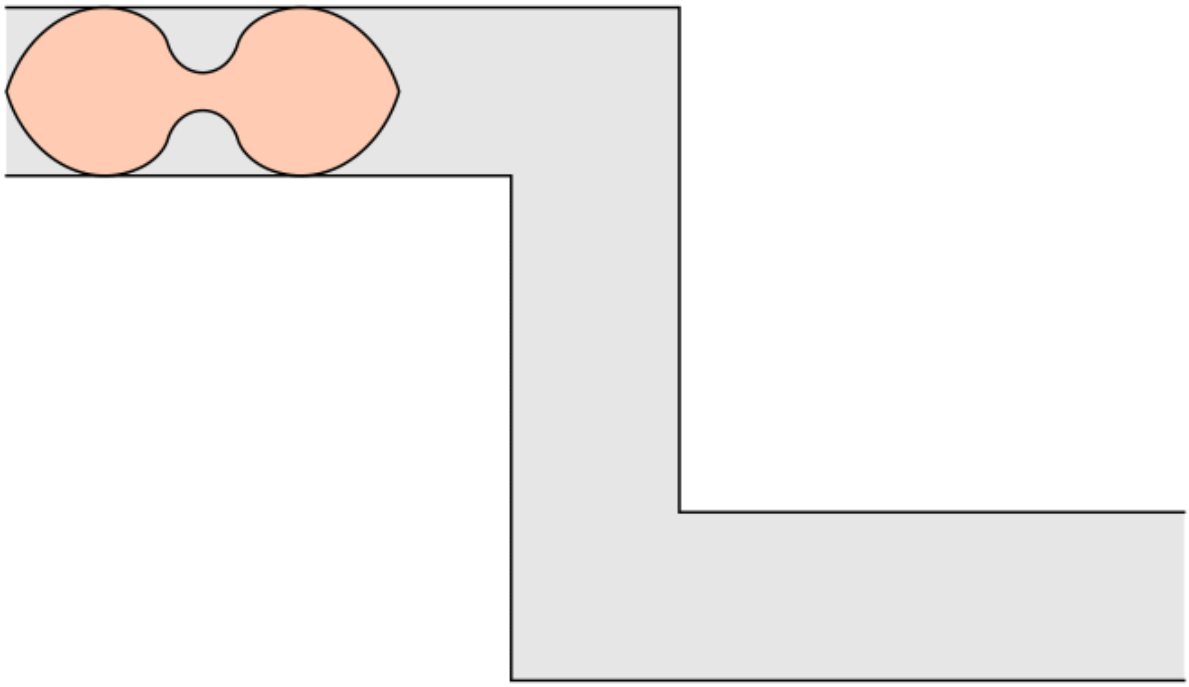
New twist on sofa problem that stumped mathematicians and furniture movers

March 20 2017, by Becky Oskin



The Moving Sofa problem asks, what is the largest shape that can move around a right-angled turn? UC Davis mathematician Dan Romik has extended this problem to a hallway with two turns, and shows that a 'bikini top' shaped sofa is the largest so far found that can move down such a hallway. Credit: Dan Romik, UC Davis

Most of us have struggled with the mathematical puzzle known as the "moving sofa problem." It poses a deceptively simple question: What is

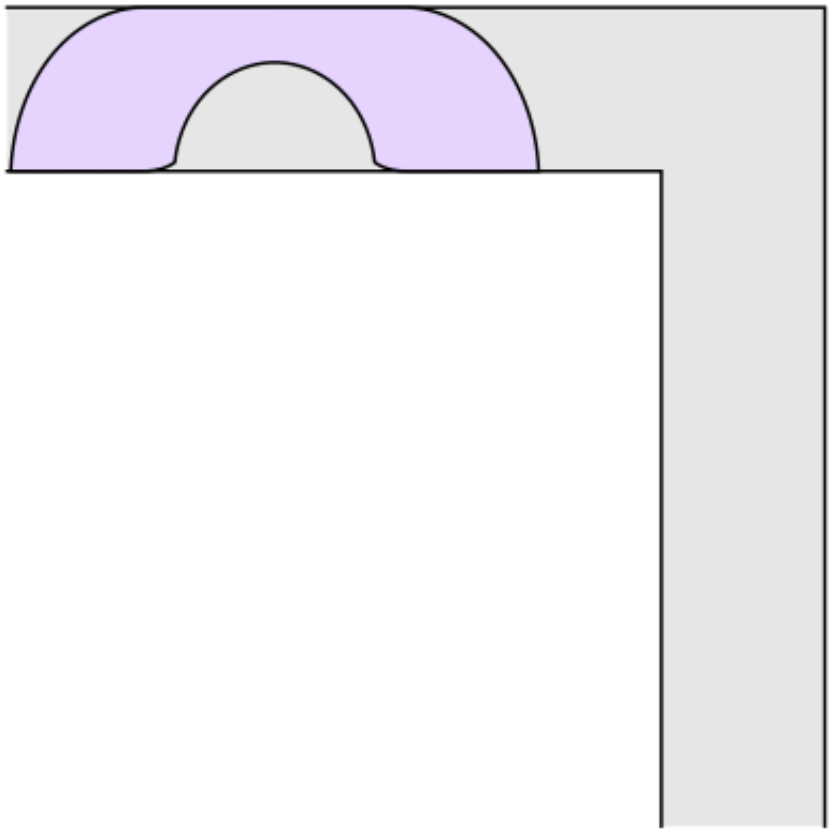
the largest sofa that can pivot around an L-shaped hallway corner?

A mover will tell you to just stand the sofa on end. But imagine the sofa is impossible to lift, squish or tilt. Although it still seems easy to solve, the moving sofa problem has stymied math sleuths for more than 50 years. That's because the challenge for mathematicians is both finding the largest sofa and proving it to be the largest. Without a proof, it's always possible someone will come along with a better solution.

"It's a surprisingly tough problem," said math professor Dan Romik, chair of the Department of Mathematics at UC Davis. "It's so simple you can explain it to a child in five minutes, but no one has found a proof yet.

The largest area that will fit around a corner is called the "sofa constant" (yes, really). It is measured in units where one unit corresponds to the width of the hallway.

Inspired by his passion for 3-D printing, Romik recently tackled a twist on the sofa problem called the ambidextrous moving sofa. In this scenario, the sofa must maneuver around both left and right 90-degree turns. His findings are published online and will appear in the journal *Experimental Mathematics*.



The Gerver sofa is the largest found that will fit round a single turn. It has a "sofa constant" of 2.22 units, where one unit represents the width of the hallway. Credit: Dan Romik/UC Davis

Eureka Moment

Romik, who specializes in combinatorics, enjoys pondering tough questions about shapes and structures. But it was a hobby that sparked Romik's interest in the moving sofa problem—he wanted to 3-D print a sofa and hallway. "I'm excited by how 3-D technology can be used in math," said Romik, who has a 3-D printer at home. "Having something you can move around with your hands can really help your intuition."

The Gerver sofa—which resembles an old telephone handset—is the

biggest sofa found to date for a one-turn hallway. As Romik tinkered with translating Gerver's equations into something a 3-D printer can understand, he became engrossed in the mathematics underlying Gerver's solution. Romik ended up devoting several months to developing new equations and writing computer code that refined and extended Gerver's ideas. "All this time I did not think I was doing research. I was just playing around," he said. "Then, in January 2016, I had to put this aside for a few months. When I went back to the program in April, I had a lightbulb flash. Maybe the methods I used for the Gerver sofa could be used for something else."

Romik decided to tackle the problem of a hallway with two turns. When tasked with fitting a sofa through the hallway corners, Romik's software spit out a shape resembling a bikini top, with symmetrical curves joined by a narrow center. "I remember sitting in a café when I saw this new shape for the first time," Romik said. "It was such a beautiful moment."

Finding Symmetry

Like the Gerver sofa, Romik's ambidextrous sofa is still only a best guess. But Romik's findings show the question can still lead to new mathematical insights. "Although the moving sofa problem may appear abstract, the solution involves new mathematical techniques that can pave the way to more complex ideas," Romik said. "There's still lots to discover in math."

More information: Dan Romik, Differential Equations and Exact Solutions in the Moving Sofa Problem, *Experimental Mathematics* (2017). DOI: 10.1080/10586458.2016.1270858

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