Differential equations and the role of coincidence

March 13 2017, by Ingrid Ladner

In a project funded by the Austrian Science Fund FWF, a team directed by mathematician Erika Hausenblas is investigating how the pressure of flowing liquids can be described when taking into account random perturbations.

Differential equations are a physicist's most important tool. From Newton's law of gravitation to quantum physics one will scarcely find a natural law expressed without differential equations describing the changes over time of a certain property of a system. If one knows the way a property changes, one can predict its future value with precision – at least in theory.

Incooperating coincidence

In reality, however, there are always random disruptions that distort the outcome. In some situations, such as flowing liquids, even minute disturbances can change the system's behaviour so radically that it makes sense to integrate an element of coincidence (or stochastic element) in the equations from the start. Such "stochastic partial differential equations" are the speciality of Erika Hausenblas, a professor of applied mathematics at the Leoben University of Mining Sciences (Montanuniversität Leoben). In a project supported by the Austrian Science Fund FWF, Hausenblas is investigating the stochastic version of "Navier-Stokes equations" which describe the motion of fluid substances and gases. As an indispensable element of engineering ever since they were first laid down in the early 19th century, these equations are put to use in a wide range of fields, from the aerodynamics of aircraft wings to water flow in a pipe or blood circulation in veins. When it comes to their stochastic version, however, there are some gaps that scientists still do not fully comprehend.

"This is a relatively young field", says Erika Hausenblas. "The first book on stochastic partial differential equations was published in the 1990s. The aim is not to eliminate coincidence, but to integrate it in the computations and describe it." Hence such equations do not have exact solutions, but give a probability distribution, and their main objective is to assess levels of uncertainty. "Financial mathematics already makes intense use of such equations", notes Hausenblas.

A lack of instruments to predict pressure

The FWF project directed by Erika Hausenblas concentrates on one part of the Navier-Stokes equations that describes pressure. Knowing the pressure of fluids in motion is important, for instance, in order to understand "cavitation", the formation of short-lived gas bubbles in flowing liquids following drops in pressure. The emergence and subsequent collapse of the bubbles create strong shock waves which are the most frequent cause of damage in water pipes or turbines. "When it comes to fluid pressure, we lack both simulation methods and methods to assess errors", explains Hausenblas. As the first researcher to focus on these questions, the mathematician underlines their great practical relevance. Meteorology is one of the fields that make intense use of Navier-Stokes equations, and pressure changes are of decisive importance in this context. So far, meteorologists have rarely used stochastic Navier-Stokes equations, although these might enable them to produce more accurate weather forecasts.

The young mathematician Tsiry Randrianasolo is participating in the project as a doctoral student. He is a graduate of the African Institute for Mathematical Sciences (AIMS) which offers a Master's programme at six universities on the African continent. The project is designed to run until November. First results have already been submitted for publication and further publications are in the offing.

The benefits of uncertainty

Hausenblas expects it will take several years for the new methods to actually be applied in engineering. In her view, this would presuppose a change in attitudes. Stochastic differential equations are not universally popular, due to a perceived lack of clarity in their outcomes. Many people try to reduce the probability of errors to zero. "But there are cases when it would make more sense to include an element of uncertainty in the computation."

In this context, the mathematician underlines how important it is for such basic research projects to be funded by the FWF. Its financial support enabled her to establish her own research group on these specialised issues.

More information: Analytic Properties of Markov Semigroup Generated by Stochastic Differential Equations Driven by Lévy Processes. Potential Anal. 46 (2017) DOI: 10.1007/s11118-016-9570-1

Provided by Austrian Science Fund (FWF)

Citation: Differential equations and the role of coincidence (2017, March 13) retrieved 24 April 2024 from https://phys.org/news/2017-03-differential-equations-role-coincidence.html