Combining the past and future of a change process in fractional calculus

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A raindrop is a sphere. It may not look like it, what with its pointy head and rounded bottom, but it is. Fractional calculus can prove it.

Fractional calculus is concerned not only with how quickly and to what extent change develops, but also in what order it advances. This is math with memory. It can be used to recall the past, perhaps to see if the raindrop was once a sphere. It can also predict the future, to determine, for instance, if the raindrop will become a sphere.

A team of mathematicians from the University of Averio, in Portugal, combined the two operators—the past and future of the change process—into one theory. They published their work in *IEEE/CAA Journal of Automatica Sinica* (*JAS*).

"Many real-world phenomena are better described by derivatives," said Dr.Ricardo Alemida, an assistant professor at the University of Averio in Portugal and an author on the paper, referencing the mathematical term for measuring the sensitivity of a variable to changing circumstances. "In fact, fractional derivatives have unique characteristics that may model certain dynamics more efficiently."

The field began with questions concerning such things as whether a raindrop contains the volume required to make a sphere. The answers resulted in the ability to measure a variable's past reaction to change, as well as predict future reactions, laying the groundwork for a number of technologies and applications.

Reading an article online, receiving a text, taking a picture, listening to the radio—none of these things would be possible without accounting for how each variable changed or will change.

Compare a letter to an email. It doesn't matter if a letter is torn or delivered to the wrong address. The letter inherently remains what it always was—ink on paper. An email is more complicated. The sender's program must be able not only to understand what the sender constructed, but it must also predict the environment to which it's being sent. Despite programming advancements such as text-only options or mobile reactivity, emails containing a broken image or that are difficult to read on a mobile device are common.

Dr.Almeida and his colleagues will continue to expand their work and test the stability of their combined theory by incorporating more variables. Work in this field may lead to future improvements for how we exchange information, among other applications.

"[We plan to] generalize the previous results to higher-order derivatives," said Dr.Almeida.

More information: Dina Tavares et al, Constrained fractional variational problems of variable order, *IEEE/CAA Journal of Automatica Sinica* (2017). DOI: 10.1109/JAS.2017.7510331

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