A friend of a friend is... a dense network

December 1 2016

A new theoretical model shows that networks evolve very differently depending on how often friend-of-a-friend connections occur. Credit: Pixabay

It's a familiar request in the digital age: one of your friends on social media has a friend who wants to be your friend. Frequent linking among friends of friends can cause a rapid increase in social network connectivity.

A new theoretical model shows that networks evolve very differently depending on how often these "second neighbor" connections occur. The work could offer a better understanding of how dense networks form.

Networks—like those based on social media or internet connections—are often characterized by their degree, which is the number of links per member, or node. Previous models of networks have tended to focus on sparse networks in which the degree remains finite as a network grows.

By including friend-of-friend interactions in their model, Renaud Lambiotte (University of Namur, Belgium), Paul Krapivsky (Boston University), and Uttam Bhat and Sid Redner (both Santa Fe Institute) could control the link density of the network.

"It's an incredibly simple model that can produce both sparse and dense networks," says Redner, a Santa Fe Institute professor.

In their recent paper published in *Physical Review Letters*, the researchers constructed a general network evolution in which every new node links to one target node already in the network, as well as to each of the neighbors of the target (that is, friends of friends), with copying probability p. The likelihood of each of these "copying" steps turns out to be the crucial factor in how the network evolves.

If copying is unlikely, the network evolves into a sparse, skeleton-like framework. But when the copying probability is greater than 1/2, the network becomes dense, with the number of links growing faster than the network itself. This "densifying" behavior has been observed in real world data, such as research paper citation lists, internet router maps, and other networks.

The researchers also investigated multiple-node connections, such as triangles that consist of three mutually-linked nodes. They found that the triangle count grew faster than the network for a copying probability greater than 2/3. In fact, they discovered an unlimited number of these growth transitions related to copying.

"It's kind of exotic, but cool, that such a generic model has all these transitions in it," Redner says.

If similar transitions are identified as real networks evolve—like those in social media—the model's copying mechanism could be an allegory for many real friend-of-friend interactions. The model may also offer a way to study the role of triangles and other so-called "cliques" as information or diseases spread in a population.

More information: R. Lambiotte et al, Structural Transitions in Densifying Networks, *Physical Review Letters* (2016). DOI: 10.1103/PhysRevLett.117.218301

Provided by Santa Fe Institute