Getting the most out of fractional models

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Machines make our lives easier in many ways. Whether it's a smart thermostat that learns when to turn the heat on or automatic brakes, machines traffic in the language of classical calculus. Classical calculus is good enough to capture the basic features of biological or mechanical systems and even human behavior, but it paints a grainy picture. To provide a richer description of these systems, experts are turning to fractional calculus.

Mathematical models built from this exotic but more general form of calculus come pre-installed with a way of accounting for past events. This feature allows them to mimic the memory-like effects observed in real systems, such as the stock market, communications networks, and, of course, the human brain. But before fractional models are uploaded into our devices, researchers have to be sure not only that they're complex enough to reflect real processes, but that they're not too complex to render our devices unstable and therefore useless. Mathematically speaking, they have to ensure that a system that deviates from its rest state—room temperature, for instance, in the case of a thermostat—can be controlled back to that state within a reasonable amount of time.

To address this problem, a team of mathematicians asked whether such control could be achieved for equations called fractional stochastic differential inclusions. These equations describe some of the most unpredictable and noisy systems found in the real world, such as financial markets and quantum systems. They proved the existence of solutions for two forms of these equations: convex and nonconvex. In math, convex cases are typically easier to solve when looking for the best way to control a system. Nonconvex cases are trickier to solve. The ability to prove controllability in both cases is therefore a major advantage of this method.

The mathematicians tested their technique numerically on a spring-like model. Although seemingly simple, the model was built using the same type of equations the team looked at before, making its behavior highly unpredictable—but not uncontrollable. The team was able to show that the spring could theoretically be brought back to rest from any position it might adopt.

Mathematical tools such as this will likely find increasing applications as fractional models become more widespread and complex. Scientists and engineers may do well to add these to their toolkits and ensure that their designs for new devices are both highly adaptable and controllable.

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