A fractional micro-macro model for crowds of pedestrians based on fractional mean field games

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From a very young age, we're warned against shouting "Fire!" in a crowded theater. The possibility of inciting mass panic presents an obvious moral problem. But for researchers, the situation also presents an interesting mathematical problem: How do large crowds of people behave in emergency situations? While many have turned to classical physics and calculus for the answer, a recent study shows that a branch of mathematics called fractional calculus may offer a more realistic picture of crowd dynamics.

An intuitive way of modeling crowds is to think of each person as an individual particle. This approach allows researchers to use the familiar language of Newtonian physics and differential and integral calculus to describe how people behave when clustered together. On the micro scale, individual people can be assigned velocities and trajectories as they wade through the cluster to find the closest or most convenient exit. And on the macro scale, researchers can talk about how the density or concentration of people varies over time.

Although useful in teasing out some of the simpler drivers of crowd behavior, the problem with this model is that people are much more complex than bouncing particles. We have unique thoughts and feelings that influence how we behave in large groups. Someone can choose to let us go ahead of them at a concert or completely block our path. And, mathematically speaking, these paths typically don't conform to the smooth trajectories handled by traditional calculus.

This is where fractional calculus comes in. A generalized form of traditional integral calculus, fractional calculus, comes preinstalled with a way to account for long-range interactions among particles or people. Each object in a fractional order model is given a memory that persists much longer than the short-lived interactions among particles. In other words, people do more than bounce off one another as they make their way toward an exit. They remember past interactions and negotiate to carve out the best route, often in highly complex, zigzag patterns. Fractional calculus therefore provides a much more realistic picture of crowd behavior.

Researchers modeled people as unique, cost-minimizing agents who navigate cooperatively with one another or competitively against each other, depending on the circumstances. Simulations showed that a crowd in a confined area tends to spread out and fill the space much faster in the fractional framework than in the traditional framework. This finding is in line with how people actually behave in crowded situations. Similarly, in an emergency exit scenario involving six people, researchers saw that pedestrians interacted with one another to reach a consensus before splitting up toward the exit.

Further comparison with real-world data is needed to understand the limitations of this new approach. But by offering researchers more freedom to define what motivates individual people in crowds, fractional calculus may be able to improve how firefighters and police officers handle disasters and emergencies.

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