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# 40-year math mystery and four generations of figuring: Proof of Kelmans-Seymour Conjecture in Graph Theory

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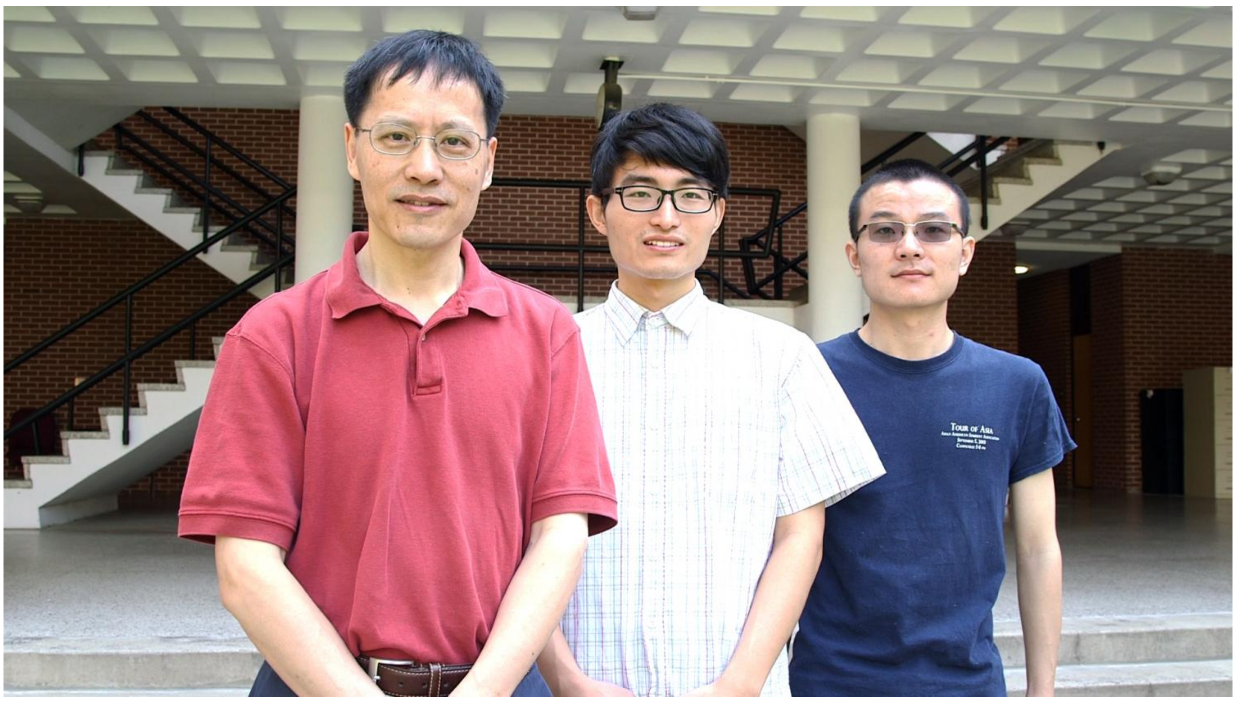

Georgia Tech mathematicians Xingxing Yu, Yan Wang and Dawei He have offered a proof of the Kelmans-Seymour Conjecture nearly 40 years after Princeton Mathematician Paul Seymour made it in 1977. Credit: Micah Eavenson / Georgia Tech

This may sound like a familiar kind of riddle: How many brilliant mathematicians does it take to come up with and prove the Kelmans-Seymour Conjecture?

But the answer is no joke, because arriving at it took mental toil that spanned four decades until this year, when mathematicians at the Georgia Institute of Technology finally announced a proof of that conjecture in Graph Theory.

Their research was funded by the National Science Foundation.

Graph Theory is a field of mathematics that's instrumental in complex tangles. It helps you make more connecting flights, helps get your GPS unstuck in traffic, and helps manage your Facebook posts.

Back to the question. How many? Six (at least).

One made the conjecture. One tried for years to prove it and failed but passed on his insights. One advanced the mathematical basis for 10 more years. One helped that person solve part of the proof. And two more finally helped him complete the rest of the proof.

Elapsed time: 39 years.

So, what is the Kelmans-Seymour Conjecture, anyway? Its name comes from Paul Seymour from Princeton University, who came up with the notion in 1977. Then another mathematician named Alexander Kelmans, arrived at the same conjecture in 1979.

And though the Georgia Tech proof fills some 120 pages of math reasoning, the conjecture itself is only one short sentence:

If a graph G is 5-connected and non-planar, then G has a TK5.

## The devil called 'TK5'

You could call a TK5 the devil in the details. TK5s are larger relatives of K5, a very simple formation that looks like a 5-point star fenced in by a pentagon. It resembles an occult or Anarchy symbol, and that's fitting. A TK5 in a "graph" is guaranteed to thwart any nice, neat "planar" status.

Graph Theory. Planar. Non-planar. TK5. Let's go to the real world to understand them better.

"Graph Theory is used, for example, in designing microprocessors and the logic behind computer programs," said Georgia Tech mathematician Xingxing Yu, who has shepherded the Kelmans-Seymour Conjecture's proof to completion. "It's helpful in detailed networks to get quick solutions that are reasonable and require low computational complexity."

To picture a graph, draw some cities as points on a whiteboard and lines representing interstate highways connecting them.

But the resulting drawings are not geometrical figures like squares and trapezoids. Instead, the lines, called "edges," are like wires connecting points called "vertices." For a planar graph, there is always some way to draw it so that the lines from point to point do not cross.

In the real world, a microprocessor is sending electrons from point to point down myriad conductive paths. Get them crossed, and the processor shorts out.

In such intricate scenarios, optimizing connections is key. Graphs and graph algorithms play a role in modeling them. "You want to get as close to planar as you can in these situations," Yu said.

In Graph Theory, wherever K5 or its sprawling relatives TK5s show up, you can forget planar. That's why it's important to know where one may be hiding in a very large graph.

## The human connections

The human connections that led to the proof of the Kelmans-Seymour Conjecture are equally interesting, if less complicated.

Seymour had a collaborator, Robin Thomas, a Regent's Professor at Georgia Tech who heads a program that includes a concentration on Graph Theory. His team has a track record of cracking decades-old math problems. One was even more than a century old.

"I tried moderately hard to prove the Kelmans-Seymour conjecture in the 1990s, but failed," Thomas said. "Yu is a rare mathematician, and this shows it. I'm delighted that he pushed the proof to completion."

Yu, once Thomas' postdoc and now a professor at the School of Mathematics, picked up on the conjecture many years later.

"Around 2000, I was working on related concepts and around 2007, I became convinced that I was ready to work on that conjecture," Yu said. He planned to involve graduate students but waited a year. "I needed to have a clearer plan of how to proceed. Otherwise, it would have been too risky," Yu said.

Then he brought in graduate student Jie Ma in 2008, and together they proved the conjecture part of the way.

Two years later, Yu brought graduate students Yan Wang and Dawei He into the picture. "Wang worked very hard and efficiently full time on the problem," Yu said. The team delivered the rest of the proof quicker than anticipated and currently have two submitted papers and two more in the works.

In addition to the six mathematicians who made and proved the conjecture, others tried but didn't complete the proof but left behind useful cues.

Nearly four decades after Seymour had his idea, the fight for its proof is still not over. Other researchers are now called to tear at it for about two years like an invading mob. Not until they've thoroughly failed to destroy it, will the proof officially stand.

Seymour's first reaction to news of the proof reflected that reality. "Congratulations! (If it's true...)," he wrote.

Graduate student Wang is not terribly worried. "We spent lots and lots of our time trying to wreck it ourselves and couldn't, so I hope things will be fine," he said.

If so, the conjecture will get a new name: Kelmans-Seymour Conjecture Proved by He, Wang and Yu.

And it will trigger a mathematical chain reaction, automatically confirming a past conjecture, Dirac's Conjecture Proved by Mader, and also putting within reach proof of another conjecture, Hajos' Conjecture.

For Princeton mathematician Seymour, it's nice to see an intuition he held so strongly is now likely to enter into the realm of proven mathematics.

"Sometimes you conjecture some pretty thing, and it's just wrong, and the truth is just a mess," he wrote in an email message. "But sometimes, the pretty thing is also the truth; that that does happen sometimes is basically what keeps math going I suppose. There's a profound thought."

Provided by Georgia Institute of Technology

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