A mathematical advance in describing waves

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New development builds on centuries of research devoted to using math to describe the physical world

One of the great joys in mathematics is the ability to use it to describe phenomena seen in the physical world, says University at Buffalo mathematician Gino Biondini.

With UB postdoctoral researcher Dionyssios Mantzavinos, Biondini has published a new paper that advances the art—or shall we say, the math—of describing a wave. The findings, published Jan. 27 in *Physical Review Letters*, are thought to apply to wave forms ranging from light waves in optical fibers to water waves in the sea.

The study explores what happens when a regular wave pattern has small irregularities, a question that scientists have been trying to answer for the last 50 years.

Researchers have long known that in many cases such minor imperfections grow and eventually completely distort the original wave as it travels over long distances, a phenomenon known as "modulational instability." But the UB team has added to this story by showing, mathematically, that many different kinds of disturbances evolve to produce wave forms belonging to a single class, denoted by their identical asymptotic state.

"Ever since Isaac Newton used math to describe gravity, applied mathematicians have been inventing new mathematics or using existing forms to describe natural phenomena," says Biondini, a professor of mathematics in the UB College of Arts and Sciences and an adjunct faculty member in the UB physics department. "Our research is, in a way, an extension of all the work that's come before."

He says the first great success in using math to represent waves came in the 1700s. The so-called wave equation, used to describe the propagation of waves such as light, sound and water waves, was discovered by Jean le Rond d'Alembert in the middle of that century. But the model has limitations.

"The wave equation is a great first approximation, but it breaks down when the waves are very large—or, in technical parlance—'nonlinear,'" Biondini said. "So, for example, in optical fibers, the wave equation is great for moderate distances, but if you send a laser pulse (which is an electromagnetic wave) through an optical fiber across the ocean or the continental U.S., the wave equation is not a good approximation anymore. "Similarly, when a water wave whitecaps and overturns, the wave equation is not a good description of the physics anymore."

Over the next 250 years, scientists and mathematicians continued to develop new and better ways to describe waves. One of the models that researchers derived in the middle of the 20th century is the nonlinear Schrödinger equation, which helps to characterize wave trains in a variety of physical contexts, including in nonlinear optics and in deep water.

But many questions remained unanswered, including what happens when a wave has small imperfections at its origin.

This is the topic of Biondini and Mantzavinos' new paper.

"Modulational instability has been known since the 1960s. When you have small perturbations at the input, you'll have big changes at the output. But is there a way to describe precisely what happens?" Biondini said. "After laying out the foundations in two earlier papers, it took us a year of work to obtain a mathematical description of the solutions. We then used computers to test whether our math was correct, and the simulation results were pretty good—it appears that we have captured the essence of the phenomenon."

The next step, Biondini said, is to partner with experimental researchers to see if the theoretical findings hold when applied to tangible, physical waves. He has started to collaborate with research groups in optics as well as water waves, and he hopes that it will soon be possible to test the theoretical predictions with real experiments.

More information: Gino Biondini et al. Universal Nature of the Nonlinear Stage of Modulational Instability, *Physical Review Letters* (2016). DOI: 10.1103/PhysRevLett.116.043902

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