Extreme numbers—the unimaginably large and small pop up in recent experiments

February 19 2016, by Kevin Knudson, University Of Florida

It’s a lot of grains of sand, but numbers can get a whole lot bigger…. Credit: Tony Hisgett, CC BY

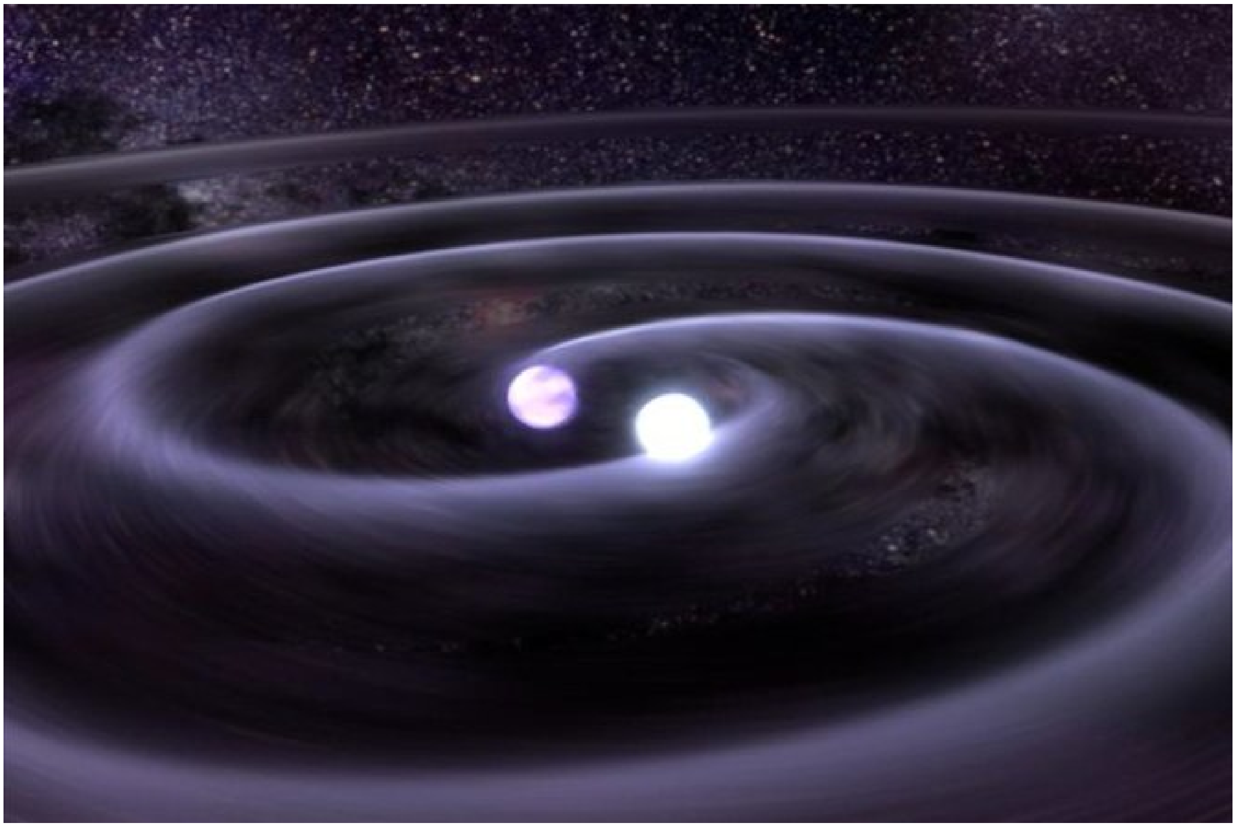
The physics world erupted in celebration this month with the confirmed discovery of gravitational waves by the Laser Interferometer

Gravitational-Wave Observatory (LIGO) group. Predicted by Einstein a century ago, the discovery verifies his description of the universe in which space and time can warp and bend.

And what is the evidence gathered by LIGO? A billion years ago, a pair of black holes of masses about 30 times that of the sun collided, releasing about three solar masses' worth of energy in the form of gravitational waves. Those waves traveled through space and reached the LIGO antennas, one in Louisiana and one in Washington, seven milliseconds apart, vibrating the mirrors at the end of each antenna's 2.5-mile-long vacuum tube by a mere four thousandths the diameter of a proton.

I'm no physicist, but the LIGO numbers intrigue me. In fact, I've noticed quite a few huge (and tiny) numbers in recently announced scientific advances, which got me to thinking about how real physical situations force us to deal with numbers so extreme they're inconceivable.

Let's unpack these numbers. The gravitational waves travel at the speed of light, so the black holes that generated them were roughly one billion light-years away from Earth. That's more than 6 billion trillion miles, or 6×10^{21} miles. The energy that created the waves is roughly equivalent to the light output of a billion trillion suns. And the end result was nudging a pair of mirrors by 4×10^{-18} meters, an unfathomably small distance.



Orbiting black holes generate gravitational waves. Credit: NASA

A great visualization of these scales can be seen in the classic film *Powers of Ten*, created by Charles and Ray Eames in 1977. It doesn't go out as far as the black holes that created the gravitational waves, nor does it go as small as the movement of the LIGO mirrors, but it does give a great sense of the scales involved in the recent announcement.

Even larger numbers

Here's a question: say you have 128 tennis balls. How many different ways can you arrange them so that each ball touches at least one other? You can stack them, lay them out in various grids, stack the layers and so on. There are probably a lot of configurations, right?

This question was answered recently by a team of researchers at Cambridge University. The number of possible arrangements is on the order of 10^{250}; that's a 1 with 250 zeroes after it. To give a sense of how large this number is, note that there are only about 10^{80} atoms in the universe. In fact, if we packed the known universe with protons, there would be only about 10^{126} of them. So if we could somehow encode each configuration of the tennis balls on an atom (or even a subatomic particle), we would be able to get through only about the cube root of the total number of possibilities.

Since it's impossible to actually count all the arrangements of the balls, the team used an indirect approach. They took a sample of all the possible configurations and computed the probability of each of them occurring. Extrapolating from there, the team was able to deduce the number of ways the entire system could be arranged, and how one ordering was related to the next. The latter is the so-called configurational entropy of the system, a measure of how disordered the particles in a system are.

This may seem like an odd calculation to make, but it is an important question in granular physics. This is the study of the behavior of materials that are granular in nature, such as sand or snow. If we wish to understand how sand dunes form and evolve over time, or how avalanches happen, we must first be able to enumerate the possible initial configurations of the particles. Clearly, 128 particles is nowhere near a large enough number for us to begin to understand a sand dune, but it's a start. And the methods employed for this study may yield insights that will help attack bigger systems.

Still bigger numbers

A number such as 10^{250} is enormous, but relative to numbers "close" to infinity it is effectively zero. At scales like this, I find it comforting to turn to literature and philosophy. In "The Library of Babel," the fascinating short story by Jorge Luis Borges, we learn about a certain library in which each book has 410 pages, and each page has 40 lines of 80 characters. The alphabet in use has 22 letters and three punctuation marks, making a total of 25 orthographic characters. We are told that every possible book is somewhere in this imagined library. So, how many books are there? First note that there are 410 x 40 x 80 = 1,312,000 characters in each book and since we have 25 choices for each character, there are $25^{1312000}$ possible books. As a power of 10, that's roughly $10^{1834097}$.

How many ways can tennis balls be arranged? Credit: Atomic Taco, CC BY-SA

If we can't wrap our heads around 10^{250}, how are we to manage a number like this? Borges' fictional library tells us how. While we can't possibly enumerate a catalog of all the books, we can imagine any book we like. There is a completely blank book. There is a book with a single comma in the middle of page 204 and nothing else. There are actually 1,312,000 books with a single comma and nothing else (just in each of the possible locations). There is a book with only the letter y in every spot. This article you're reading right now appears exactly as it is written (by spelling out the numbers and ignoring extraneous punctuation) in an enormous number of books in the library (10 to a very large power, certainly more than 1.7 million). It appears in every language on the planet (suitably translated into the alphabet).

If you want to play around with this idea, there is an online Library of Babel that catalogs every possible page of 3200 characters. This amounts to only about 10^{4677} books, a tiny fraction of the total library, but it's great fun to search for strings of characters. Jonathan Basile, the site's creator, has devised a scheme for cataloging the books based on Borges' description of the library as a collection of hexagonal cells with a certain number of books on each shelf (only four of each cell's six walls contain shelves). For example, the phrase "when in the course of human events" occurs by itself at the top of page 186 of volume 21 on shelf 1 of wall 3 of a hexagon labeled with a 3254-digit identifier in base 36. Whew.

And yet, despite the enormity of the Library of Babel, the number of books is less than the largest known prime number, discovered in January 2016. The Mersenne number M74207281 = $2^{74207281}$ - 1 has more than 22 million digits, way more than the puny number of books in the library (only about 1.8 million digits). And there are surely larger primes out there (Euclid told us so), with billions, trillions, or 10^{250} digits.

Should we care?

So, are these unimaginable numbers actually good for anything? In a practical sense, no. They are simply too large to be useful in everyday scientific computation (we need big primes for encryption algorithms, but not *that* big). And once you've counted every subatomic particle in the universe, there's probably not much need for a bigger number. They do provide fertile ground for thought experiments, though, and illustrate the human capacity to ponder the unreasonably large (and small, too).

This article was originally published on The Conversation. *Read the* original article.

Source: The Conversation

Citation: Extreme numbers—the unimaginably large and small pop up in recent experiments (2016, February 19) retrieved 27 April 2024 from https://phys.org/news/2016-02-extreme-numbersthe-unimaginably-large-small.html