Corals, crochet and the cosmos: how hyperbolic geometry pervades the universe

January 28 2016, by Margaret Wertheim, University Of Melbourne

The frilly forms of corals and sponges are biological variations of hyperbolic geometry, as seen here on the Great Barrier Reef, near Cairns, Queensland, Australia. Credit: Wikimedia/Toby Hudson, CC BY-SA

We have built a world of largely straight lines – the houses we live in, the skyscrapers we work in and the streets we drive on our daily commutes. Yet outside our boxes, nature teams with frilly, crenellated forms, from the fluted surfaces of lettuces and fungi to the frilled skirts of sea slugs and the gorgeous undulations of corals.

These organisms are biological manifestations of what we call hyperbolic geometry, an alternative to the Euclidean geometry we learn about in school that involves lines, shapes and angles on a flat surface or plane. In hyperbolic geometry the plane is not necessarily so flat.

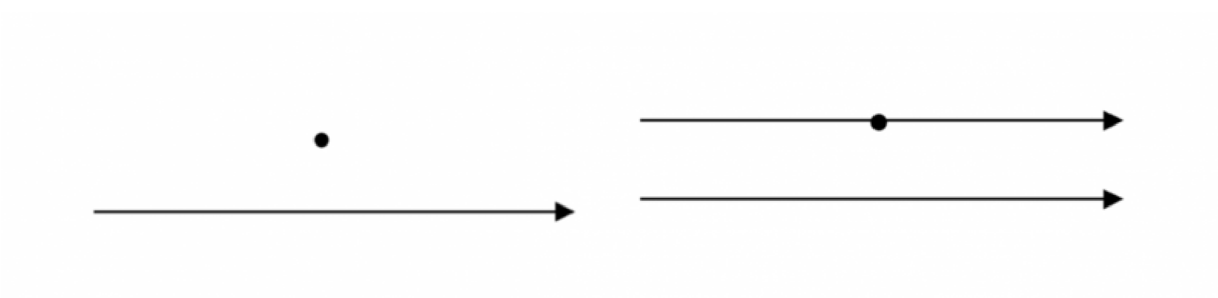
Yet while nature has been playing with hyperbolic forms for hundreds of millions of years, mathematicians spent hundreds of years trying to prove that such structures were impossible.

But these efforts led to a realisation that hyperbolic geometry is logically legitimate. And that, in turn, led to the revolution that produced the kind of maths now underlying general relativity, and thus the structure of the universe.

Non-Euclidean clause

Hyperbolic geometry is radical because it violates one of the axioms of Euclidean geometry, which long stood as a model for reason itself.

The fifth and final axiom of Euclid's system – the so-called parallel postulate – turns out not to be correct. Or at least not necessarily so. If we accept it, we get Euclidean geometry, but if we abandon it, other geometries become possible, most famously the hyperbolic variety.



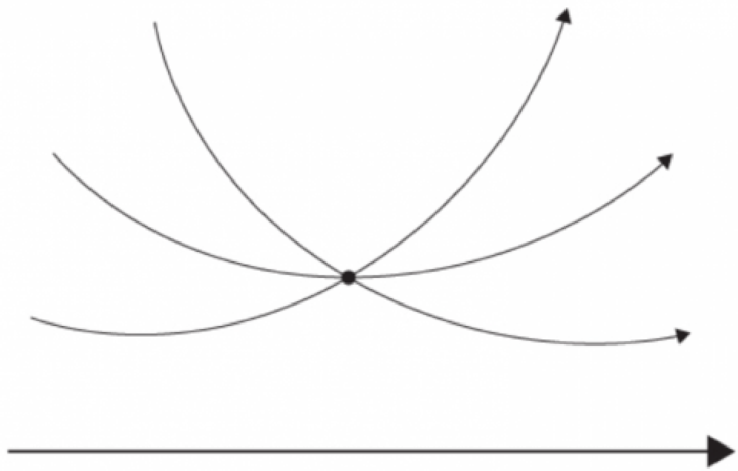
Euclid could only see one possible straight line through a point that does not meet the original line. Credit: Margaret Wertheim, Author provided

Here's how the parallel postulate works. Consider a simple question: if I have a straight line, and a point outside the line, how many straight lines can I draw through the point that never meet the original line? Euclid said the answer is *one* and there couldn't be any more, which feels intuitively right.

Mathematicians, being sticklers, wanted to prove this was true, but in the end such efforts led them to see that there is a logically consistent geometric system in which the answer is infinity. We can represent the situation as follows.

This seems impossible and a first reaction is to say it's cheating because the lines look curved. But they only look curved because we're trying to project an image of a curved surface onto a flat plane.

It's the same as when we're trying to project an image of the surface of the Earth onto a flat map; the relationships get distorted. To really see countries relative to one another we have to look at a globe.



What if the straight lines look curved? Credit: Margaret Wertheim, Author provided

So also with hyperbolic geometry. To really see what's going on we have to look at the curved surface itself, and here the lines are straight.

One way of understanding different geometries is in terms of their curvature. A flat, or Euclidean plane has zero curvature. The surface of a sphere (like a beach ball) has positive curvature, and a hyperbolic plane has negative curvature. It's a geometric analogue of a negative number.

When mathematicians discovered this aberrant geometry in the early 19th century they were nearly driven mad. "For God's sake please give it up," said the Hungarian mathematician Wolfgang Bolyai to his son János Bolyai, urging him abandon to work on hyperbolic geometry.

Nature's work

Yet critters who'd never studied non-Euclidean geometry had meanwhile just been doing it. Along with corals, many other species of reef organisms have hyperbolic forms, including sponges and kelps.

Wherever there is an advantage to maximising surface area – such as for filter feeding animals – hyperbolic shapes are an excellent solution. There are hyperbolic structures in cells, hyperbolic cacti and hyperbolic flowers, such as calla lilies. In the film Avatar, there is a fabulous CGI grove of giant hyperbolic blooms that curl up when touched.

Hyperbolic surfaces can also be built at the molecular scale from carbon atoms. These carbon nano-foams were discovered in 1997 by physicist Andrei Rode and his colleagues at the Australian National University.

This image shows straight lines drawn on a paper model of a hyperbolic plane. All the pencil lines that appear to be curved were drawn with a ruler so they are actually straight. Credit: Margaret Cagyle, Institute For Figuring, Author provided

That year Cornell mathematician Daina Taimina also worked out how to model such surfaces using crochet, which was a big deal because it's actually hard for humans to construct these forms.

For the past 10 years, I've been spearheading a project where we use hyperbolic crochet to make woolly simulations of coral reefs. Our Crochet Coral Reefs are an artistic response to the devastation of living reefs due to global warming and have been exhibited at art galleries and science museums around the world, including the Smithsonian.

Here, a ball of wool and a crochet hook become pedagogical tools bringing mathematics out of textbooks, and taking it to people as a living tactile experience.

More than 8,000 women in a dozen countries (including Australia, the United States of America, and the United Arab Emirates) have participated in making these installations, which reside at the intersection of mathematics, marine biology, community art practice and environmentalism.

The shape of the universe

Once mathematicians realised that different geometrical spaces are possible, a question arose as to which one is realised in physical space. What is the shape of our universe?

Galileo Galilei and Isaac Newton founded modern physics on the assumption that space is Euclidean, but Albert Einstein's equations of general relativity describe a universe that can have complex curved forms.

One of the major questions astronomers are trying to resolve, with instruments such as the Hubble Space Telescope, is what shape our universe has. While most of the large-scale evidence points to a Euclidean structure, there is some tantalising evidence that we might just live in a hyperbolic world.

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