# Small entropy changes allow quantum measurements to be nearly reversed 

September 30 2015, by Lisa Zyga
(Phys.org)—In 1975, Swedish physicist Göran Lindblad developed a theorem that describes the change in entropy that occurs during a quantum measurement. Today, this theorem is a foundational component of quantum information theory, underlying such important concepts as the uncertainty principle, the second law of thermodynamics, and data transmission in quantum communication systems.

Now, 40 years later, physicist Mark M. Wilde, Assistant Professor at Louisiana State University, has improved this theorem in a way that allows for understanding how quantum measurements can be approximately reversed under certain circumstances. The new results allow for understanding how quantum information that has been lost during a measurement can be nearly recovered, which has potential implications for a variety of quantum technologies.

## Quantum relative entropy never increases

Most people are familiar with entropy as a measure of disorder and the law that "entropy never decreases"-it either increases or stays the same during a thermodynamic process, according to the second law of thermodynamics. However, here the focus is on "quantum relative entropy," which in some sense is the negative of entropy, so the reverse is true: quantum relative entropy never increases, but instead only decreases or stays the same.

In fact, this was the entropy inequality theorem that Lindblad proved in 1975: that the quantum relative entropy cannot increase after a measurement. In this context, quantum relative entropy is interpreted as a measure of how well one can distinguish between two quantum states, so it's this distinguishability that can never increase. (Wilde describes a proof of Lindblad's result in greater detail in his textbook Quantum Information Theory, published by Cambridge University Press.)

One thing that Lindblad's proof doesn't address, however, is whether it makes any difference if the quantum relative entropy decreases by a little or by a lot after a measurement.

In the new paper, Wilde has shown that, if the quantum relative entropy decreases by only a little, then the quantum measurement (or any other type of so-called "quantum physical evolution") can be approximately reversed.
"When looking at Lindblad's entropy inequality, a natural question is to wonder what we could say if the quantum relative entropy goes down only by a little when the quantum physical evolution is applied," Wilde told Phys.org. "It is quite reasonable to suspect that we might be able to approximately reverse the evolution. This was arguably open since the work of Lindblad in 1975, addressed in an important way by Denes Petz in the late 1980s (for the case in which the quantum relative entropy stays the same under the action of the evolution), and finally formulated as a conjecture around 2008 by Andreas Winter. What my work did was to prove this result as a theorem: if the quantum relative entropy goes down only by a little under a quantum physical evolution, then we can approximately reverse its action."

## Wide implications

Wilde's improvements to Lindblad's theorem have a variety of
implications, but the main one that Wilde discusses in his paper is how the new results allow for recovering quantum information.
"If the decrease in quantum relative entropy between two quantum states after a quantum physical evolution is relatively small," he said, "then it is possible to perform a recovery operation, such that one can perfectly recover one state while approximately recovering the other. This can be interpreted as quantifying how well one can reverse a quantum physical evolution." So the smaller the relative entropy decrease, the better the reversal process.

The ability to recover quantum information could prove useful for quantum error correction, which aims to protect quantum information from damaging external effects. Wilde plans to address this application more in the future with his colleagues.

As Wilde explained, Lindblad's original theorem can also be used to prove the uncertainty principle of quantum mechanics in terms of entropies, as well as the second law of thermodynamics for quantum systems, so the new results have implications in these areas, as well.
"Lindblad's entropy inequality underlies many limiting statements, in some cases said to be physical laws or principles," Wilde said. "Examples are the uncertainty principle and the second law of thermodynamics. Another example is that this entropy inequality is the core step in determining limitations on how much data we can communicate over quantum communication channels. We could go as far as to say that the above entropy inequality constitutes a fundamental law of quantum information theory, which is a direct mathematical consequence of the postulates of quantum mechanics."

Regarding the uncertainty principle, Wilde and two coauthors, Mario Berta and Stephanie Wehner, discuss this angle in a forthcoming paper.

They explain that the uncertainty principle involves quantum measurements, which are a type of quantum physical evolution and therefore subject to Lindblad's theorem. In one formulation of the uncertainty principle, two experiments are performed on different copies of the same quantum state, with both experimental outcomes having some uncertainty.
"The uncertainty principle is the statement that you cannot generally make the uncertainties of both experiments arbitrarily small, i.e., there is generally a limitation," Wilde said. "It is now known that a statement of the uncertainty principle in terms of entropies can be proved by using the 'decrease of quantum relative entropy inequality.' So what the new theorem allows for doing is relating the uncertainties of the measurement outcomes to how well we could try to reverse the action of one of the measurements. That is, there is now a single mathematical inequality which captures all of these notions."

In terms of the second law of thermodynamics, Wilde explains how the new results have implications for reversing thermodynamic processes in both classical and quantum systems.
"The new theorem allows for quantifying how well we can approximately reverse a thermodynamic transition from one state to another without using any energy at all," he said.

He explained that this is possible due to the connection between entropy, energy, and work. According to the second law of thermodynamics, a thermodynamic transition from one quantum state to another is allowed only if the free energy decreases from the original state to the final state. During this process, one can gain work and store energy. This law can be rewritten as a statement involving relative entropies and can be proved as a consequence of the decrease of quantum relative entropy.
"What my new work with Stephanie Wehner and Mischa Woods allows for is a refinement of this statement," Wilde said. "We can say that if the free energy does not go down by very much under a thermodynamic transition (i.e., if there is not too much work gained in the process), then it is possible to go back approximately to the original state from the final state, without investing any work at all. The key word here is that you can go back only approximately, so we are not in violation of the second law, only providing a refinement of it."

In addition to these implications, the new theorem can also be applied to other research topics in quantum information theory, including the Holevo bound, quantum discord, and multipartite information measures.

Wilde's work was funded in part by The DARPA Quiness program (ending now), which focused on quantum key distribution, or using quantum mechanics to ensure secret communication between two parties. He describes more about this application, in particular how Alice and Bob might use a quantum state to share secrets that can be kept private from an eavesdropper Eve (and help them survive being attacked by a bear), in a recent blog post.

More information: Göran Lindblad. "Completely positive maps and entropy inequalities." Communications in Mathematical Physics. June 1975, Volume 40, Issue 2, pp 147-151

Mark M. Wilde. "Recoverability in quantum information theory." Proceedings of the Royal Society A. DOI: 10.1098/rspa.2015.0338 Also at arXiv:1505.04661 [quant-ph]

Stephanie Wehner, Mark M. Wilde, and Mischa P. Woods. "Work and reversibility in quantum thermodynamics." arXiv:1506.08145 [quant-ph]

In a very recent preprint at arXiv:1509.07127 [quant-ph], Wilde and his
colleagues have further improved upon the result from the paper in the Proceedings of the Roayl Society A, which should allow for it to be applied even more widely and potentially allow for experimental tests.

## © 2015 Phys.org

Citation: Small entropy changes allow quantum measurements to be nearly reversed (2015, September 30) retrieved 20 March 2024 from https://phys.org/news/2015-09-small-entropy-quantum-reversed.html

This document is subject to copyright. Apart from any fair dealing for the purpose of private study or research, no part may be reproduced without the written permission. The content is provided for information purposes only.

