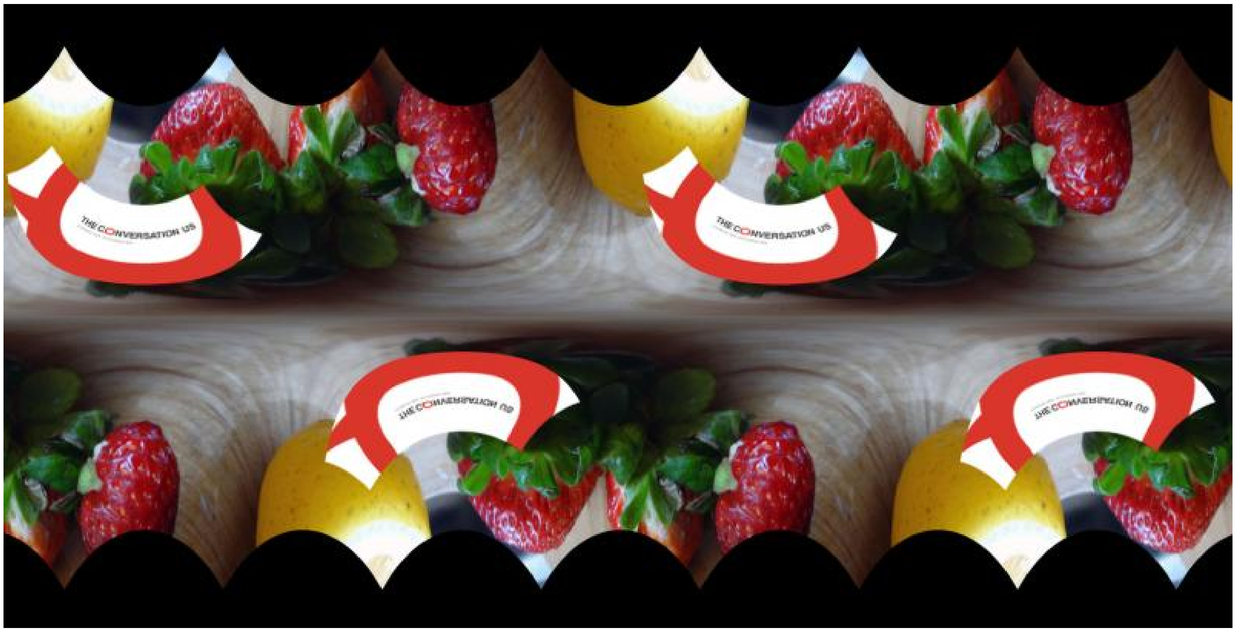
Patterns are math we love to look at

September 22 2015, by Frank A Farris

Repeating patterns are visually intriguing. Credit: Frank A Farris, CC BY-ND

Why do humans love to look at patterns? I can only guess, but I've written a whole book about new mathematical ways to make them. In Creating Symmetry, The Artful Mathematics of Wallpaper Patterns, I include a comprehensive set of recipes for turning photographs into patterns. The official definition of "pattern" is cumbersome; but you can think of a pattern as an image that repeats in some way, perhaps when we rotate, perhaps when we jump one unit along.

Here's a pattern I made, using the logo of The Conversation, along with some strawberries and a lemon:



Repeating forever left and right. Credit: Frank A Farris, CC BY-ND

Mathematicians call this a frieze pattern because it repeats over and over again left and right. Your mind leads you to believe that this pattern repeats indefinitely in either direction; somehow you know how to continue the pattern beyond the frame. You also can see that the pattern along the bottom of the image is the same as the pattern along the top, only flipped and slid over a bit.

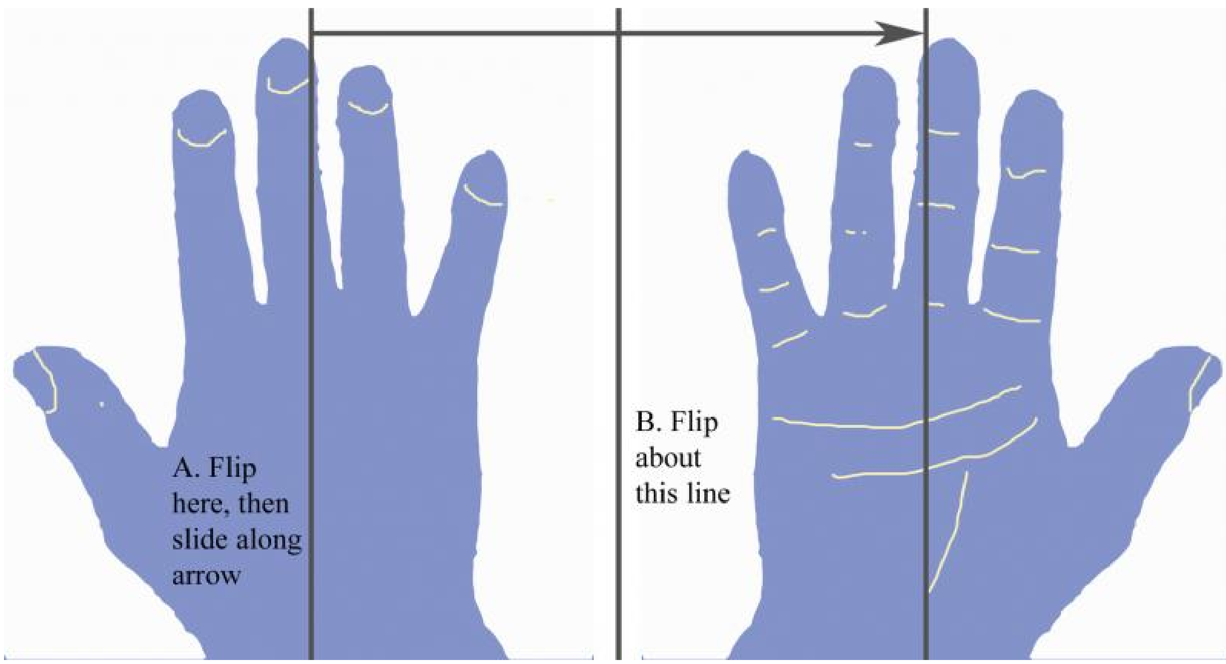
When we can do something to a pattern that leaves it unchanged, we call that a symmetry of the pattern. So sliding this pattern sideways just the right amount – let's call that translation by one unit – is a symmetry of my pattern. The flip-and-slide motion is called a glide reflection, so we say the above pattern has glide symmetry.

You can make frieze patterns from rows of letters, as long as you can imagine that the row continues indefinitely left and right. I'll indicate that idea by …AAAAA…. This row of letters definitely has what we call translational symmetry, since we can slide along the row, one A at a time, and wind up with the same pattern.

What other symmetries does it have? If you use a different font for your A's, that could mess up the symmetry, but if the legs of the letter A are the same, as above, then this row has reflection symmetry about a vertical axis drawn through the center of each A.

Now here's where some interesting mathematics comes in: did you notice the reflection axis between the As? It turns out that every frieze pattern with one vertical mirror axis, and hence an infinite row of them (by the translational symmetry shared by all friezes), must necessarily have an additional set of vertical mirror axes exactly halfway between the others. And the mathematical explanation is not too hard.

Suppose a pattern stays the same when you flip it about a mirror axis. And suppose the same pattern is preserved if you slide it one unit to the right. If doing the first motion leaves the pattern alone and doing the second motion also leaves the pattern alone, then doing first one and then the other leaves the pattern alone.



Flipping and then sliding is the same as one big flip. Credit: Frank A Farris, CC BY-ND

You can act this out with your hand: put your right hand face down on a table with the mirror axis through your middle finger. First flip your hand over (the mirror symmetry), then slide it one unit to the right (the translation). Observe that this is exactly the same motion as flipping your hand about an axis half a unit from the first.

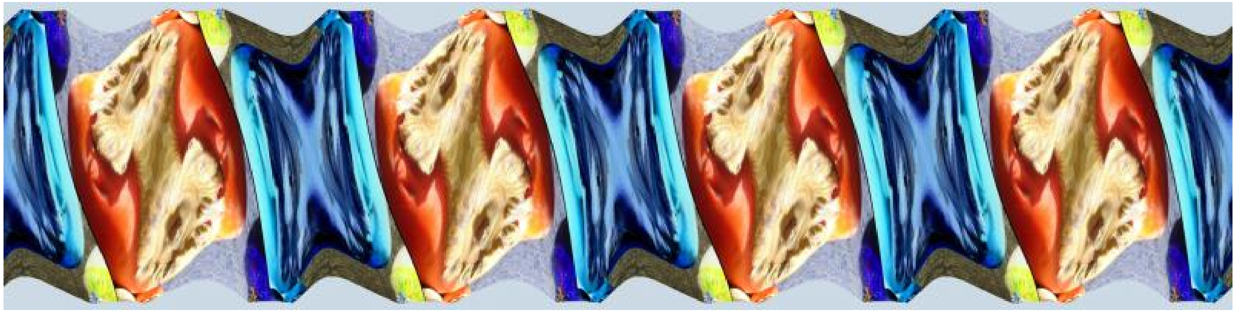
That proves it! No one can create a pattern with translational symmetry and mirrors without also creating those intermediate mirror symmetries. This is the essence of the mathematical concept of group: if a pattern has some symmetries, then it must have all the others that arise from combining those.

The surprising thing is that there are only a few different types of frieze symmetry. When I talk about types, I mean that a row of A's has the same type as a row of V's. (Look for those intermediate mirror axes!) Mathematicians say that the two groups of symmetries are isomorphic, meaning of the same form.

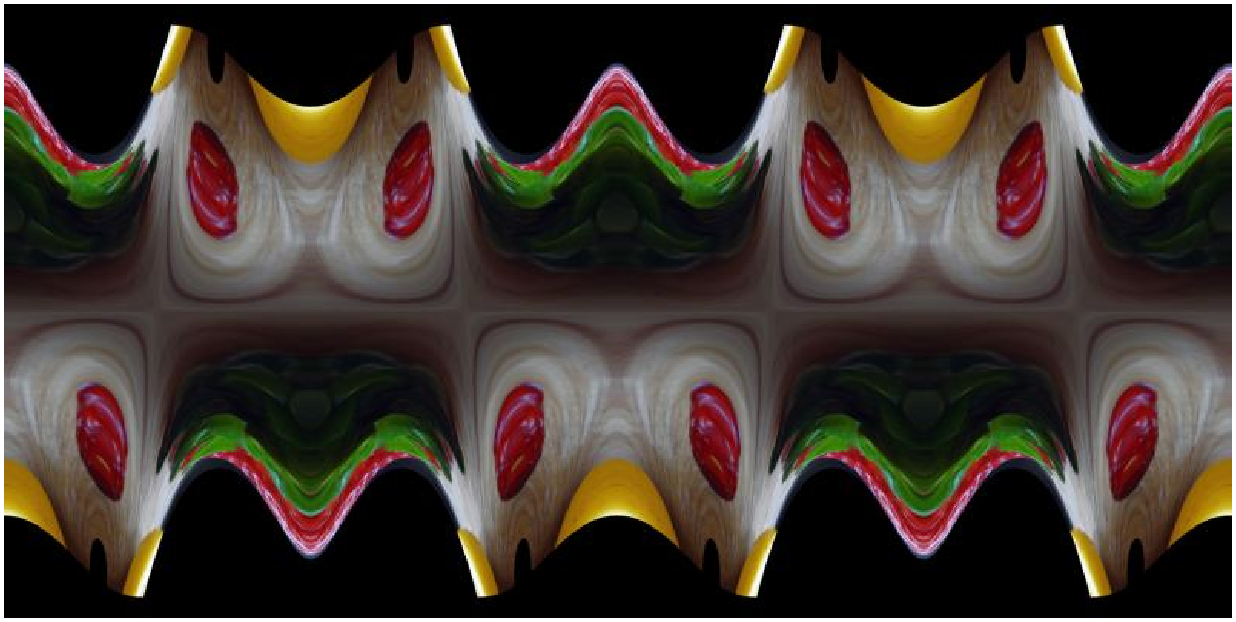
It turns out there are exactly seven different frieze groups. Surprised? You can probably figure out what they are, with some help. Let me explain how to name them, according to the International Union of Crystallographers.

The naming symbol uses the template prvh, where the p is just a placeholder, the r denotes rotational symmetry (think of a row of N's), the v marks vertical qualities and the h is for horizontal. The name for the pattern of A's is p1m1: no rotation, vertical mirror, no horizontal feature beyond translation. They use 1 as a placeholder when that particular kind of symmetry does not occur in the pattern.

What do I mean by horizontal stuff? My introductory frieze was p11g, because there's glide symmetry in the horizontal directions and no symmetry in the other slots.



Write down a bunch of rows of letters and see what types of symmetry you can name. Hint: the persimmon pattern above (or that row of N's) would be named p211. There can't be a p1g1 because we insist that our frieze has translational symmetry in the horizontal direction. There can't be a p1mg because if you have the m in the vertical direction and a g in the horizontal, you're forced (not by me, but by the nature of reality) to have rotational symmetry, which lands you in p2mg.



It's hard to make p2mg patterns with letters, so here's one made from the same lemon and strawberries. I left out the logo, as the words became too distorted. Look for the horizontal glides, vertical mirrors, and centers of twofold rotational symmetry. (Here's a funny feature: the smiling strawberry faces turn sad when you see them upside down.)

In my book, I focus more on wallpaper patterns: those that repeat forever along two different axes. I explain how to use mathematical formulas called complex wave forms to construct wallpaper patterns. I prove that every wallpaper group is isomorphic – a mathematical concept meaning of the same form – to one of only 17 prototype groups. Since pattern types limit the possible structures of crystals and even atoms, all results of this type say something deep about the nature of reality.

Ancient Roman mosaic floor in Carranque, Spain. Credit: a_marga, CC BY-SA

Whatever the adaptive reasons for our human love for patterns, we have been making them for a long time. Every decorative tradition includes the same limited set of pattern types, though sometimes there are cultural reasons for breaking symmetry or omitting certain types. Did our visual love for recognizing that "Yes, this is the same as that!" originally have a useful root, perhaps evolving from an advantage in distinguishing edible from poisonous plants, for instance? Or do we just like them? Whyever it is, we still get pleasure from these repetitive patterns tens of thousands of years later.

Source: The Conversation

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