Physicists propose new definition of time crystals—then prove such things don't exist

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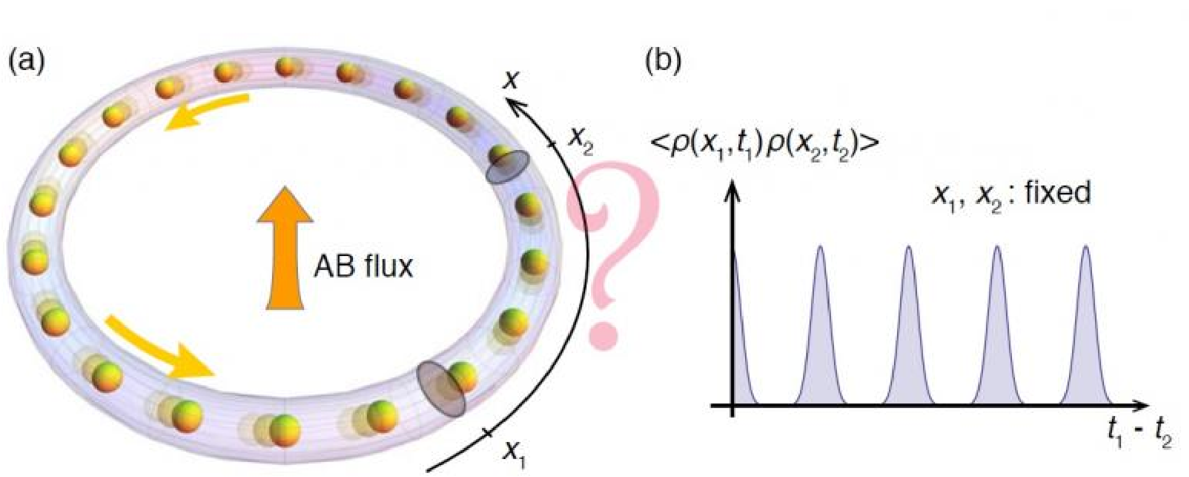

Figure showing a possible realization of time crystals, (a) consisting of ions in a ring and (b) involving oscillatory behavior. Credit: Watanabe and Oshikawa. ©2015 American Physical Society

(Phys.org)—For the past few years, physicists have been intrigued by a hypothetical system called a "quantum time crystal," which has the unusual property of exhibiting periodic motion in its ground state, which is its state of lowest energy. This behavior is unexpected, as it suggests that the system can move even when it doesn't seem to have enough energy to do so. Ever since time crystals were first proposed in 2012 by Frank Wilczek, physicists have raised serious doubts over their existence, with a few studies already disproving their existence in specific cases.

Although at this point it seems likely that time crystals are merely hypothetical concepts, physicists want to be as certain as possible that they do not exist before permanently laying the idea to rest.

With this motivation, physicists Haruki Watanabe at the University of California at Berkeley and Masaki Oshikawa at the University of Tokyo have published a paper in *Physical Review Letters* in which they first propose a precise definition of time crystals, and then prove a no-go theorem that rules out the possibility of their existence when defined in this way.

"The original proposal of Wilczek was about a ground state, and we proved its impossibility quite generally," Watanabe told *Phys.org*. "We also managed to extend the argument for a ground state (at zero temperature) to a more general 'thermal equilibrium' state (at non-zero temperatures). These two situations are what we usually discuss for spontaneous symmetry breaking. Thus, we would like to stress that, in our opinion, the spontaneous breaking of time translation symmetry in the standard sense has been proven impossible. In this respect, we believe that our result pretty much settles the debate."

Yet, as the physicists explain more below, a few open questions still remain.

Time crystals and ordinary crystals

Because a precise mathematical definition of time crystals has been lacking until now, it has understandably been difficult to settle the question of their existence. Watanabe and Oshikawa's general definition of time crystals resembles the definition of ordinary crystals. Both definitions are based on "long-range order," which refers to a crystal's characteristic structure of a highly ordered pattern that repeats over long distances. In ordinary crystals, such as diamonds, long-range order accounts for the periodic geometric shape. In time crystals, according to the new definition, long-range order describes the oscillating motion over time.

When investigating whether time crystals defined in this way actually exist, Watanabe and Oshikawa explain that the key criteria is that the term "crystal" should be reserved for systems that exhibit correlated, coherent behavior. However, due to thermodynamic reasons, they show that a system cannot exhibit this type of behavior at any temperature, and so no real system can be considered a time crystal by this definition.

Overall, the results show that, while ordinary crystals can and do exist, time crystals cannot exist. The only difference between the two is that the first involves spatial long-range order while the second involves temporal long-range order. Although space and time are often considered to be closely related, together forming the "fabric of spacetime," the findings here emphasize a subtle yet fundamental difference between space and time—with a result that allows for the existence of crystals in one dimension, but forbids them in the other.

Time crystals in equilibrium

There is one minor caveat to the physicists' definition of time crystals: it requires that the system exist in a state of equilibrium, which is basically the state that a system acquires after a long time without any external forces acting on it. It's well-known that some non-equilibrium systems exhibit spontaneous oscillations, as demonstrated by a swinging pendulum. However, the researchers explain that these systems should not be considered time crystals without further justification.

"The oscillation of a pendulum or the quantum oscillation in the AC

Josephson effect have been known for a long time, and they are not surprising because they are not in thermal equilibrium," Oshikawa explained. "Certainly they should not be regarded as time crystals. This is a trivial example of periodic motion seen when the system is not in a ground state or in thermal equilibrium. The motion of the pendulum with a small amplitude can be described by a harmonic oscillator, and essentially the same behavior can occur in quantum and classical systems. … Ordinary crystals, on the other hand, can certainly be realized in equilibrium at low enough temperatures."

With that being said, the scientists also explained that it may still be possible to realize true time crystals in a state of non-equilibrium, but doing so would require formulating a new definition and answering several controversial questions.

"Although it is in the textbooks that any state will approach thermal equilibrium after leaving it for a long enough time—and this has been supported in countless numbers of experiments—this is a highly nontrivial statement and is still under active study," Watanabe said. "Even if it is true after an infinitely long time, if it requires an unrealistically long time (for example, longer than the age of the universe) to reach the equilibrium, it would be rather irrelevant for actual observation.

"We may generalize the definition of time crystals to non-thermal/non-equilibrium setups, and we expect more works along this line. For example, we can easily come up with two interesting open questions one can ask: First, can excited eigenstates [quantum mechanical states] show a time-dependent long range order? This question is related to a recent hot topic, the so-called 'many-body localization.' Second, can discrete time translation be spontaneously broken? The continuous time translation cannot be spontaneously broken as we explained above. However, when a system is periodically driven (this makes the problem non-equilibrium), the relevant symmetry is discrete time translation.

"Time crystals may be realized in some way in a non-equilibrium setup, but we would need an appropriate definition for that (as we explained above, there are many systems that show periodic motion rather trivially and we should not include them). As far as we are aware of, there is no positive evidence suggesting such a possibility so far, however."

More information: Haruki Watanabe and Masaki Oshikawa. "Absence of Quantum Time Crystals." *Physical Review Letters*. DOI: 10.1103/PhysRevLett.114.251603
Also at: arXiv:1410.2143 **[cond-mat.stat-mech]**